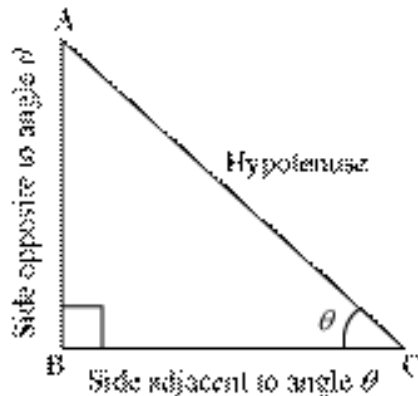


Trigonometrical Identities

- Trigonometric Ratio



$$\sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

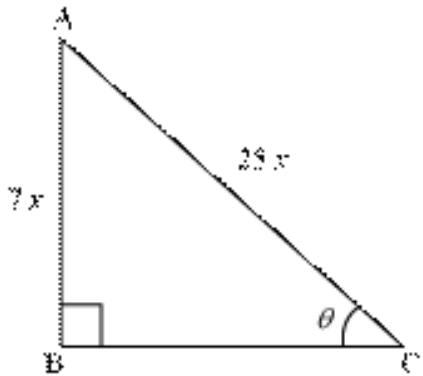
$$\text{Also, } \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of the angle can be calculated.

Example:

If $\sin \theta = \frac{7}{25}$, then find the value of $\sec \theta(1 + \tan \theta)$.

Solution:



It is given that $\sin \theta = \frac{7}{25}$

$$\sin \theta = \frac{AB}{AC} = \frac{7}{25}$$

$\Rightarrow AB = 7x$ and $AC = 25x$, where x is some positive integer

By applying Pythagoras theorem in $\triangle ABC$, we get:

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow (7x)^2 + BC^2 = (25x)^2$$

$$\Rightarrow 49x^2 + BC^2 = 625x^2$$

$$\Rightarrow BC^2 = 625x^2 - 49x^2$$

$$\Rightarrow BC = \sqrt{576}x = 24x$$

$$\therefore \sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{25}{24}$$

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{7}{24}$$

$$\therefore \sec \theta (1 + \tan \theta) = \frac{25}{24} \left(1 + \frac{7}{24} \right) = \frac{25}{24} \times \frac{31}{24} = \frac{775}{576}$$

- **Trigonometric Identities**

1. $\cos^2 A + \sin^2 A = 1$

2. $1 + \tan^2 A = \sec^2 A$

3. $1 + \cot^2 A = \operatorname{cosec}^2 A$

Example:

If $\cos \theta = \frac{5}{7}$, find the value of $\cot \theta + \operatorname{cosec} \theta$

Solution:

We have, $\cos \theta = \frac{5}{7}$

$$\text{Now, } \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \left(\frac{5}{7}\right)^2}$$

$$= \sqrt{\frac{49-25}{49}} = \frac{2\sqrt{6}}{7}$$

$$\therefore \operatorname{cosec} \theta = \frac{7}{2\sqrt{6}}$$

$$\text{Also, } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = \frac{5}{2\sqrt{6}}$$

$$\therefore \cot \theta + \operatorname{cosec} \theta = \frac{5}{2\sqrt{6}} + \frac{7}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \sqrt{6}$$

- **Trigonometric Ratios of Complementary Angles**

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

$$\cot(90^\circ - \theta) = \tan \theta$$

$$\operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

Where θ is an acute angle.

Example 1: Evaluate the expression

$$\sin 28^\circ \sin 30^\circ \sin 54^\circ \sec 36^\circ \sec 62^\circ$$

Solution:

$$\sin 28^\circ \sin 30^\circ \sin 54^\circ \sec 36^\circ \sec 62^\circ$$

$$= (\sin 28^\circ \sec 62^\circ) (\sin 54^\circ \sec 36^\circ) \sin 30^\circ$$

$$= \{\sin 28^\circ \operatorname{cosec}(90^\circ - 62^\circ)\} \{\sin 54^\circ \operatorname{cosec}(90^\circ - 36^\circ)\} \sin 30^\circ$$

$$= (\sin 28^\circ \operatorname{cosec} 28^\circ) (\sin 54^\circ \operatorname{cosec} 54^\circ) \sin 30^\circ$$

$$= \left(\sin 28^\circ \frac{1}{\sin 28^\circ} \right) \left(\sin 54^\circ \frac{1}{\sin 54^\circ} \right) \times \frac{1}{2}$$

$$= \frac{1}{2}$$

Example 2: Evaluate the expression

$$4\sqrt{3}(\sin 40^\circ \sec 30^\circ \sec 50^\circ) + \frac{\sin^2 34^\circ + \sin^2 56^\circ}{\sec^2 31^\circ - \cot^2 59^\circ}$$

Solution:

$$\begin{aligned}
& 4\sqrt{3}(\sin 40^\circ \sec 30^\circ \sec 50^\circ) + \frac{\sin^2 34^\circ + \sin^2 56^\circ}{\sec^2 31^\circ - \cot^2 59^\circ} \\
&= 4\sqrt{3}[\sec 30^\circ(\sin 40^\circ \sec 50^\circ)] + \frac{\sin^2 34^\circ + \sin^2(90^\circ - 56^\circ)}{\sec^2 31^\circ - \tan^2(90^\circ - 59^\circ)} \\
&\quad [\because \cos(90^\circ - \theta) = \sin \theta, \tan(90^\circ - \theta) = \cot \theta] \\
&= 4\sqrt{3}[\sec 30^\circ \sin 40^\circ \operatorname{cosec}(90^\circ - 50^\circ)] + \frac{\sin^2 34^\circ + \cos^2 34^\circ}{\sec^2 31^\circ - \tan^2 31^\circ} \\
&= 4\sqrt{3}\left[\frac{2}{\sqrt{3}} \sin 40^\circ \operatorname{cosec} 40^\circ\right] + \frac{1}{1} \\
&= 8 + 1 = 9
\end{aligned}$$

- If we know the sine, cosine and tangent values for θ lying between 0° and 90° , then we can find the trigonometric ratio values for all other angles using trigonometric identities and complementary angle identities.
- To find the sine, cosine and tangent values of all angles between 0° and 90° , we use the trigonometric tables.

For example, to find the value of $\sin 29^\circ 44'$, we read the table of natural sines in the horizontal line, which begins with 29° , and in the vertical column headed by $42'$.

$$\sin 29^\circ 42' = 0.4955$$

Now, we read, in the same horizontal line, the value written in the mean difference column headed by $2'$ ($44' = 42' + 2'$).

It is found to be 5.

As θ increases, sine value increases.

$\therefore$ The mean difference value is to be added.

$$\begin{aligned}
\therefore \sin 29^\circ 44' &= \sin(29^\circ 42' + 2') \\
&= 0.4955 + 0.0005 = 0.4960
\end{aligned}$$