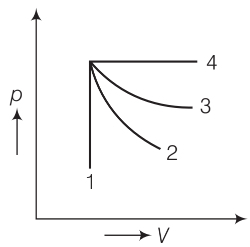
11

Thermodynamics

Multiple Choice Questions (MCQs)

Q. 1 An ideal gas undergoes four different processes from the same initial state (figure). Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3 and 4 which one is adiabatic?

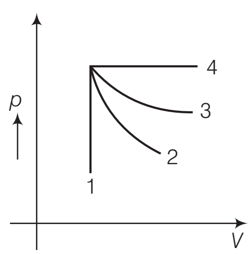


(a) 4 (b) 3 (c) 2 (d) 1

Thinking Process

The slope of the curve for the adiabatic process will be more that is the curve will be steeper. Slope of p-V curve in adiabatic process = $\gamma\,(p/V)$ where as slope of is otemal process = $-p/v$

Ans. *(c)* For the curve 4 pressure is constant, so this is an isobaric process.



For the curve 1, volume is constant, so it is isochoric process. Between curves 3 and 2, curve 2 is steeper, so it is adiabatic and 3 is isothermal.

Note *We should be careful while deciding between isothermal and adiabatic curves because these curves look similar.*

Q. 2 If an average person jogs, he produces 14.5×10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580×10^3 cal for evaporation) is

(a) 0.25 kg (b) 2.25 kg (c) 0.05 kg (d) 0.20 kg

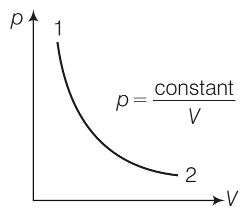
Ans. *(a)* Amount of sweat evaporated/minute

$$= \frac{\text{Sweat produced / minute}}{\text{Number of calories required for evaporation / kg}}$$

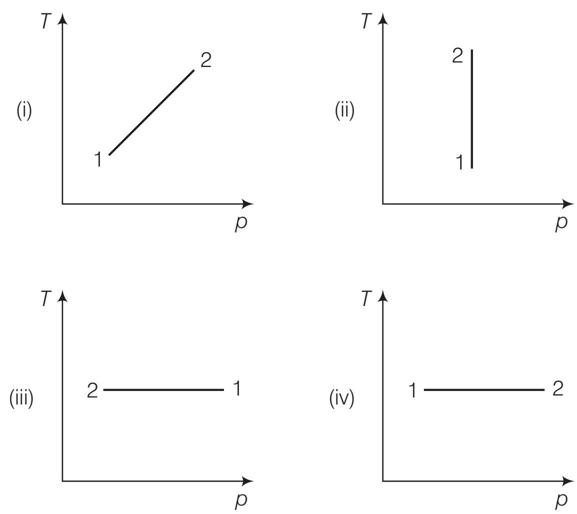
$$= \frac{\text{Amount of heat produced per minute in jogging}}{\text{Latent heat (in cal / kg)}}$$

$$= \frac{14.5 \times 10^3}{580 \times 10^3} = \frac{145}{580} = 0.25 \text{ kg}$$

Q. 3 Consider *p-V* diagram for an ideal gas shown in figure.



Out of the following diagrams, which figure represents the *T-p* diagram?



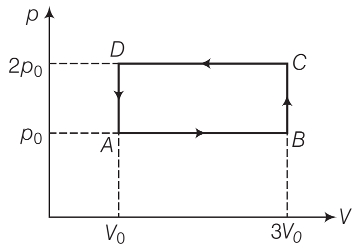
(a) (iv) (b) (ii) (c) (iii) (d) (i)

Ans. *(c)* According to the question given that pV = constant

Hence, we can say that the gas is going through an isothermal process.

Clearly, from the graph that between process 1 and 2 temperature is constant and the gas expands and pressure decreases *i.e.,* $p_2 < p_1$ which corresponds to diagram (iii).

Q. 4 An ideal gas undergoes cyclic process *ABCDA* as shown in given *p-V* diagram. The amount of work done by the gas is



(a) $6p_0V_0$ (b) $-2p_0V_0$ (c) $+2p_0V_0$ (d) $+4p_0V_0$

Thinking Process

Work done in a process by which a gas is going through can be calculated by area of the p-V diagram.

Ans. *(b)* Consider the *p-V* diagram given in the question.

Work done in the process *ABCD* = area of rectangle *ABCDA*

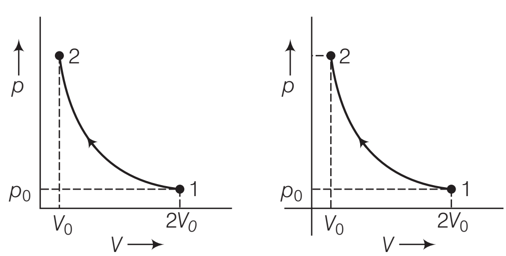
$$= (AB) \times BC = (3V_0 - V_0) \times (2p_0 - p_0)$$
$$= 2V_0 \times p_0 = 2\,p_0\,V_0$$

As the process is going anti-clockwise, hence there is a net compression in the gas. So, work done by the gas $= -2\,p_0\,V_0$.

Q. 5 Consider two containers *A* and *B* containing identical gases at the same pressure, volume and temperature. The gas in container *A* is compressed to half of its original volume isothermally while the gas in container *B* is compressed to half of its original value adiabatically. The ratio of final pressure of gas in *B* to that of gas in *A* is

(a) $2^{\gamma-1}$ (b) $\left(\frac{1}{2}\right)^{\gamma-1}$ (c) $\left(\frac{1}{1-\gamma}\right)^2$ (d) $\left(\frac{1}{\gamma-1}\right)^2$

Ans. *(a)* Consider the *p-V* diagram shown for the container *A* (isothermal) and for container *B* (adiabatic).



Container *A* (Isothermal) Container *B* (Adiabatic)

Both the process involving compression of the gas.

For isothermal process (gas *A*) (during $1 \rightarrow 2$)

$$p_1V_1 = p_2V_2$$
$$\Rightarrow \quad p_0\,(2\,V_0) = p_2\,(V_0)$$
$$\Rightarrow \quad p_2 = 2p_0$$

For adiabatic process, (gas B) (during $1 \rightarrow 2$)

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\Rightarrow \quad p_0 (2V_0)^\gamma = p_2 (V_0)^\gamma$$

$$\Rightarrow \quad p_2 = \left(\frac{2V_0}{V_0}\right)^\gamma p_0 = (2)^\gamma p_0$$

Hence, $\frac{(p_2)_B}{(p_2)_A}$ = Ratio of final pressure = $\frac{(2)^\gamma p_0}{2p_0} = 2^{\gamma - 1}$

where, γ is ratio of specific heat capacities for the gas.

Q. 6 Three copper blocks of masses M_1, M_2 and M_3 kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at T_1, T_2, T_3 $(T_1 > T_2 > T_3)$. Assuming there is no heat loss to the surroundings, the equilibrium temperature T is (s is specific heat of copper)

(a) $T = \frac{T_1 + T_2 + T_3}{3}$

(b) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

(c) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$

(d) $T = \frac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

Ans. ***(b)*** Let the equilibrium temperature of the system is T.

Let us assume that $T_1, T_2 < T < T_3$.

According to question, there is no net loss to the surroundings.

Heat lost by M_3 = Heat gained by M_1 + Heat gained by M_2

$$\Rightarrow \quad M_3 s(T_3 - T) = M_1 s(T - T_1) + M_2 s(T - T_2)$$

(where, s is specific heat of the copper material)

$$\Rightarrow \quad T[M_1 + M_2 + M_3] = M_3T_3 + M_1T_1 + M_2T_2$$

$$\Rightarrow \quad T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$$

Multiple Choice Questions (More Than One Options)

Q. 7 Which of the processes described below are irreversible?

(a) The increase in temperature of an iron rod by hammering it

(b) A gas in a small container at a temperature T_1 is brought in contact with a big reservoir at a higher temperature T_2 which increases the temperature of the gas

(c) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston

(d) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight w is added to the piston, resulting in compression of gas

Thinking Process

If any process can be returned back such that both, the system and the surroundings return to their original states, with no other change anywhere else in the universe, then this process is called reversible process.

Ans. ***(a, b, d)***

(a) When the rod is hammered, the external work is done on the rod which increases its temperature. The process cannot be retraced itself.

(b) In this process energy in the form of heat is transferred to the gas in the small container by big reservoir at temperature T_2.

(d) As the weight is added to the cylinder arrangement in the form of external pressure hence, it cannot be reversed back itself.

Q. 8 An ideal gas undergoes isothermal process from some initial state *i* to final state *f*. Choose the correct alternatives.

(a) $dU = 0$ (b) $dQ = 0$ (c) $dQ = dU$ (d) $dQ = dW$

Ans. ***(a, d)***

For an isothermal process change in temperature of the system $dT = 0 \Rightarrow T =$ constant.

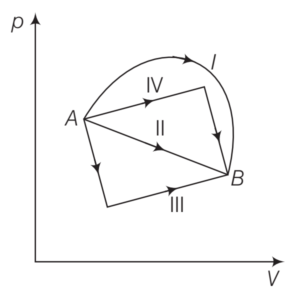
We know that for an ideal gas dU = change in internal energy $= nC_V \, dT = 0$

[where, n is number of moles and C_V is specific heat capacity at constant volume]

From first law of thermodynamics,

$$dQ = dU + dW$$
$$= 0 + dW \Rightarrow dQ = dW$$

Q. 9 Figure shows the *p-V* diagram of an ideal gas undergoing a change of state from *A* to *B*. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state.



(a) Change in internal energy is same in IV and III cases, but not in I and II

(b) Change in internal energy is same in all the four cases

(c) Work done is maximum in case I

(d) Work done is minimum in case II

Thinking Process

Internal energy is a state function and work done by the gas is a path dependent function.The work done in a thermodynamical process is equal to the area bounded between p-V curve.

Ans. ***(b, c)***

Change in internal energy for the process *A* to *B*

$$dU_{A \to B} = nC_V \, dT = nC_V(T_B - T_A)$$

which depends only on temperatures at *A* and *B*.

Work done for *A* to *B*, $dW_{A \to B}$ = Area under the *p-V* curve which is maximum for the path I.

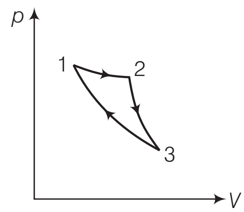
Q. 10 Consider a cycle followed by an engine (figure.)

1 to 2 is isothermal

2 to 3 is adiabatic

3 to 1 is adiabatic

Such a process does not exist, because



(a) heat is completely converted to mechanical energy in such a process, which is not possible

(b) mechanical energy is completely converted to heat in this process, which is not possible

(c) curves representing two adiabatic processes don't intersect

(d) curves representing an adiabatic process and an isothermal process don't intersect

Ans. *(a, c)*

(a) The given process is a cyclic process *i.e.,* it returns to the original state 1.

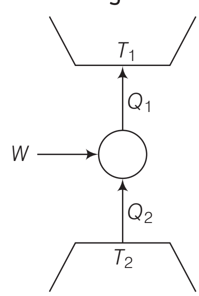
Hence, change in internal energy $dU = 0$

$$\Rightarrow \quad dQ = dU + dW = 0 + dW = dW$$

Hence, total heat supplied is converted to work done by the gas (mechanical energy) which is not possible by second law of thermodynamics.

(c) When the gas expands adiabatically from 2 to 3. It is not possible to return to the same state without being heat supplied, hence the process 3 to 1 cannot be adiabatic.

Q. 11 Consider a heat engine as shown in figure. Q_1 and Q_2 are heat added both to T_1 and heat taken from T_2 in one cycle of engine. W is the mechanical work done on the engine.



If $W > 0$, then possibilities are

(a) $Q_1 > Q_2 > 0$

(b) $Q_2 > Q_1 > 0$

(c) $Q_2 < Q_1 < 0$

(d) $Q_1 < 0, Q_2 > 0$

Ans. *(a, c)*

Consider the figure we can write $Q_1 = W + Q_2$

$$\Rightarrow \quad W = Q_1 - Q_2 > 0 \qquad \text{(By question)}$$

$$\Rightarrow \quad Q_1 > Q_2 > 0 \qquad \text{(If both } Q_1 \text{ and } Q_2 \text{ are positive)}$$

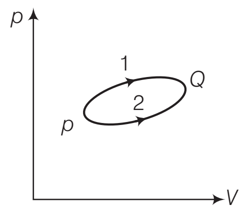
We can also, write $Q_2 < Q_1 < 0$ (If both Q_1 and Q_2 are negative).

Very Short Answer Type Questions

Q. 12 Can a system be heated and its temperature remains constant?

Ans. *Yes*, this is possible when the entire heat supplied to the system is utilised in expansion. *i.e.,* its working against the surroundings.

Q. 13 A system goes from P to Q by two different paths in the p-V diagram as shown in figure. Heat given to the system in path 1 is 1000 J. The work done by the system along path 1 is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?



Thinking Process

We have to apply first law of thermodynamics for each path.

Ans. For path 1, Heat given $Q_1 = +1000$ J

Work done $= W_1$ (let)

For path 2,

Work done $(W_2) = (W_1 - 100)$ J

Heat given $Q_2 = ?$

As change in internal energy between two states for different path is same.

$\therefore \quad \Delta U = Q_1 - W_1 = Q_2 - W_2$

$1000 - W_1 = Q_2 - (W_1 - 100)$

$\Rightarrow \quad Q_2 = 1000 - 100 = 900$ J

Q. 14 If a refrigerator's door is kept open, will the room become cool or hot? Explain.

Ans. If a refrigerator's door is kept open, then room will become hot, because amount of heat removed would be less than the amount of heat released in the room.

Q. 15 Is it possible to increase the temperature of a gas without adding heat to it? Explain.

Ans. *Yes*, during adiabatic compression the temperature of a gas increases while no heat is given to it.

In adiabatic compression, $dQ = 0$

$\therefore$ From first law of thermodynamics, $dU = dQ - dW$

$dU = -dW$

In compression work is done on the gas *i.e.,* work done is negative.

Therefore, $dU =$ Positive

Hence, internal energy of the gas increases due to which its temperature increases.

Q. 16 Air pressure in a car tyre increases during driving. Explain.

Ans. During, driving, temperature of the gas increases while its volume remains constant.

So, according to Charle's law, at constant volume (V),

Pressure (p) $\propto$ Temperature (T)

Therefore, pressure of gas increases.

Short Answer Type Questions

Q. 17 Consider a Carnot's cycle operating between $T_1 = 500$ K and $T_2 = 300$ K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.

Thinking Process

The efficiency of a Carnot's engine is $\eta = 1 - \frac{T_2}{T_1}$,

where, T_2 *is temperature of the sink and* T_1 *is temperature of the source.*

Ans. Given, temperature of the source $T_1 = 500$ K

Temperature of the sink $T_2 = 300$ K

Work done per cycle $W = 1$k J $= 1000$ J

Heat transferred to the engine per cycle $Q_1 = ?$

Efficiency of a Carnot engine $(\eta) = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{200}{500} = \frac{2}{5}$

and $\eta = \frac{W}{Q_1}$

$\Rightarrow$ $Q_1 = \frac{W}{\eta} = \frac{1000}{(2/5)} = 2500$ J

Q. 18 A person of mass 60 kg wants to lose 5kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is burnt on expending 7000 k cal, how many times must he go up and down to reduce his weight by 5 kg?

Thinking Process

Potential energy(PE) of an object at height (h) is mgh. The energy in the form of fat will be utilised to increase PE of the person. Thus, the calorie consumed by the person in going up is mgh, then according to problem calorie consumed by the person in coming down is $\frac{1}{2}mgh$.

Ans. Given, height of the stairs $= h = 10$ m

Energy produced by burning 1 kg of fat = 7000 kcal

$\therefore$ Energy produced by burning 5 kg of fat $= 5 \times 7000 = 35000$ kcal

$= 35 \times 10^6$ cal

Energy utilised in going up and down one time

$$= mgh + \frac{1}{2}mgh = \frac{3}{2}mgh$$

$$= \frac{3}{2} \times 60 \times 10 \times 10$$

$$= 9000 \text{ J} = \frac{9000}{4.2} = \frac{3000}{1.4} \text{ cal}$$

$\therefore$ Number of times, the person has to go up and down the stairs

$$= \frac{35 \times 10^6}{(3000/1.4)} = \frac{35 \times 1.4 \times 10^6}{3000}$$

$$= 16.3 \times 10^3 \text{ times}$$

Q. 19 Consider a cycle tyre being filled with air by a pump. Let V be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V(<<V)$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from p_1 to p_2?

Thinking Process

There is no exchange of heat in the process, hence this can be considered as an adiabatic process.

Ans. Let, volume is increased by ΔV and pressure is increased by Δp by an stroke.
For just before and after an stroke, we can write

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$$\Rightarrow \quad p(V + \Delta V)^\gamma = (p + \Delta p)V^\gamma \quad (\because \text{ volume is fixed})$$

$$\Rightarrow \quad pV^\gamma\left(1 + \frac{\Delta V}{V}\right)^\gamma = p\left(1 + \frac{\Delta p}{p}\right)V^\gamma$$

$$\Rightarrow \quad pV^\gamma\left(1 + \gamma\frac{\Delta V}{V}\right) \approx pV^\gamma\left(1 + \frac{\Delta p}{p}\right) \quad (\because \Delta v << v)$$

$$\Rightarrow \quad \gamma\frac{\Delta V}{V} = \frac{\Delta p}{p} \Rightarrow \Delta V = \frac{1}{\gamma}\frac{V}{p}\Delta p$$

$$\Rightarrow \quad dV = \frac{1}{\gamma}\frac{V}{p}dp$$

Hence, work done is increasing the pressure from p_1 to p_2

$$W = \int_{p_1}^{p_2} pdV = \int_{p_1}^{p_2} p \times \frac{1}{\gamma}\frac{V}{p}dp$$

$$= \frac{V}{\gamma}\int_{p_1}^{p_2} dp = \frac{V}{\gamma}(p_2 - p_1)$$

$$\Rightarrow \quad W = \frac{(p_2 - p_1)}{\gamma}V$$

Q. 20 In a refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1kW power and heat transferred from −3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

Thinking Process

The Carnot engine is the most efficient heat engine operating between two given temperature. This is why it is known as perfect engine. The efficiency of Carnot engine is

$$\eta = 1 - \frac{T_2}{T_1}.$$

Ans. Given, temperature of the source is 27°C

$$\Rightarrow \quad T_1 = (27 + 273)\text{K} = 300\text{K}$$

Temperature of sink $\quad T_2 = (-3 + 273)\text{K} = 270\text{K}$

Efficiency of a perfect heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{270}{300} = \frac{1}{10}$$

Efficiency of refrigerator is 50% of a perfect engine

$$\therefore \quad \eta' = 0.5 \times \eta = \frac{1}{2}\eta = \frac{1}{20}$$

$\therefore$ Coefficient of performance of the refrigerator

$$\beta = \frac{Q_2}{W} = \frac{1-\eta'}{\eta'}$$

$$= \frac{1-(1/20)}{(1/20)} = \frac{19/20}{1/20} = 19$$

$\Rightarrow$ $$Q_2 = \beta W = 19W \qquad \left(\because \beta = \frac{Q_2}{W}\right)$$

$$= 19 \times (1\text{kW}) = 19\,\text{kW} = 19\,\text{kJ/s}.$$

Therefore, heat is taken out of the refrigerator at a rate of 19 kJ per second.

Q. 21 If the coefficient of performance of a refrigerator is 5 and operates at the room temperature (27°C), find the temperature inside the refrigerator.

Thinking Process

Coefficient of performance (β) of a refrigerator is ratio of quantity of heat removed per cycle (Q_2) to the amount of work done on the refrigerator.

Ans. Given, coefficient of performace (β) = 5

$$T_1 = (27 + 273)\,K = 300\text{ K}, T_2 = ?$$

Coefficient of performance $(\beta) = \dfrac{T_2}{T_1 - T_2}$

$$5 = \frac{T_2}{300 - T_2} \Rightarrow 1500 - 5T_2 = T_2$$

$\Rightarrow$ $6T_2 = 1500 \Rightarrow T_2 = 250\text{K}$

$\Rightarrow$ $T_2 = (250 - 273)°\text{C} = -23°\text{C}$

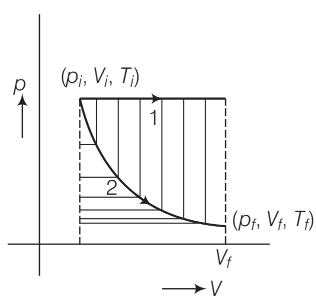
Q. 22 The initial state of a certain gas is (p_i, V_i, T_i). It undergoes expansion till its volume becomes V_f. Consider the following two cases

(a) the expansion takes place at constant temperature.

(b) the expansion takes place at constant pressure.

Plot the *p-V* diagram for each case. In which of the two cases, is the work done by the gas more?

Ans. Consider the diagram *p-V*, where variation is shown for each process.

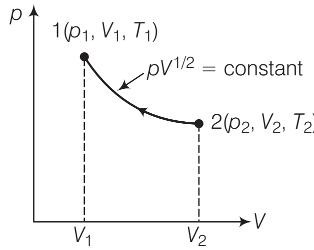


Process 1 is isobaric and process 2 is isothermal.

Since, work done = area under the *p-V* curve. Here, area under the *p-V* curve 1 is more . So, work done is more when the gas expands in isobaric process.

Long Answer Type Questions

Q. 23 Consider a p-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in figure.



(a) Find the work done when the gas is taken from state 1 to state 2.

(b) What is the ratio of temperature T_1/T_2, if $V_2 = 2V_1$?

(c) Given the internal energy for one mole of gas at temperature T is $(3/2)RT$, find the heat supplied to the gas when it is taken from state 1 to 2, with $V_2 = 2V_1$.

Ans. Let $pV^{1/2} = \text{Constant} = K, \; p = \dfrac{K}{\sqrt{V}}$

(a) Work done for the process 1 to 2,

$$W = \int_{V_1}^{V_2} p\,dV = K\int_{V_1}^{V_2} \frac{dV}{\sqrt{V}} = K\left[\frac{\sqrt{V}}{1/2}\right]_{V_1}^{V_2} = 2K(\sqrt{V_2} - \sqrt{V_1})$$

$$= 2p_1V_1^{1/2}(\sqrt{V_2} - \sqrt{V_1}) = 2p_2V_2^{1/2}(\sqrt{V_2} - \sqrt{V_1})$$

(b) From ideal gas equation,

$$pV = nRT \quad \Rightarrow \quad T = \frac{pV}{nR} = \frac{p\sqrt{V}\sqrt{V}}{nR}$$

$$\Rightarrow \quad T = \frac{K\sqrt{V}}{nR} \qquad (\text{As, } p\sqrt{V} = K)$$

Hence,

$$T_1 = \frac{K\sqrt{V_1}}{nR} \quad \Rightarrow \quad T_2 = \frac{K\sqrt{V_2}}{nR}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{\dfrac{K\sqrt{V_1}}{nR}}{\dfrac{K\sqrt{V_2}}{nR}} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V_1}{2V_1}} = \frac{1}{\sqrt{2}} \qquad (\because V_2 = 2V_1)$$

(c) Given, internal energy of the gas $= U = \left(\dfrac{3}{2}\right)RT$

$$\Delta U = U_2 - U_1 = \frac{3}{2}R(T_2 - T_1)$$

$$= \frac{3}{2}RT_1(\sqrt{V} - 1) \qquad [\because T_2 = \sqrt{2}\,T_1 \text{ from (b)}]$$

$$\Delta W = 2p_1V_1^{1/2}(\sqrt{V_2} - \sqrt{V_1})$$

$$= 2p_1V_1^{1/2}(\sqrt{2} \times \sqrt{V_1} - \sqrt{V_1})$$

$$= 2p_1V_1(\sqrt{2} - 1) = 2RT_1(\sqrt{2} - 1)$$

$$\because \quad \Delta Q = \Delta U + \Delta W$$

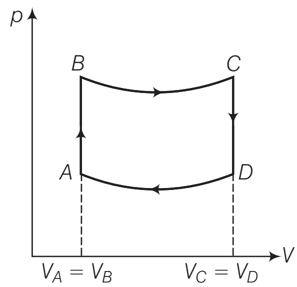
$$= \frac{3}{2}RT_1(\sqrt{2} - 1) + 2RT_1(\sqrt{2} - 1)$$

$$= (\sqrt{2} - 1)RT_1(2 + 3/2)$$

$$= \left(\frac{7}{2}\right)RT_1(\sqrt{2} - 1)$$

This is the amount of heat supplied.

Q. 24 A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.



A to *B* volume constant, *B* to *C* adiabatic, *C* to *D* volume constant and *D* to *A* adiabatic

$$V_C = V_D = 2V_A = 2V_B$$

(a) In which part of the cycle heat is supplied to the engine from outside?
(b) In which part of the cycle heat is being given to the surrounding by the engine?
(c) What is the work done by the engine in one cycle? Write your answer in term of p_A, p_B, V_A?
(d) What is the efficiency of the engine?

$$\left(\gamma = \frac{5}{3} \text{ for the gas}\right), \left(C_V = \frac{3}{2}R \text{ for one mole}\right)$$

Ans. **(a)** For the process *AB*,

$$dV = 0 \Rightarrow dW = 0 \quad (\because \text{ volume is constant})$$

$$dQ = dU + dW = dU$$

$$\Rightarrow \quad dQ = dU = \text{Change in internal energy.}$$

Hence, in this process heat supplied is utilised to increase, internal energy of the system.

Since, $p = \left(\frac{nR}{V}\right)T$, in isochoric process, $T \propto p$. So temperature increases with increases of pressure in process *AB* which inturn increases internal energy of the system *i.e.*, $dU > 0$. This imply that $dQ > 0$. So heat is supplied to the system in process *AB*.

(b) For the process *CD*, volume is constant but pressure decreases.
Hence, temperature also decreases so heat is given to surroundings.

(c) To calculate work done by the engine in one cycle, we calculate work done in each part separately.

$$W_{AB} = \int_A^B p\,dV = 0, \; W_{CD} = \int_{V_C}^{V_D} p\,dV = 0 \quad (\because dV = 0)$$

$$W_{BC} = \int_{V_B}^{V_C} p\,dV = k\int_{V_B}^{V_C} \frac{dV}{V^{\gamma}} = \frac{k}{1-\gamma}[V^{1-\gamma}]_{V_B}^{V_C}$$

$$= \frac{1}{1-\gamma}[pV]_{V_B}^{V_C} = \frac{(p_C V_C - p_B V_B)}{1-\gamma}$$

Similarly, $$W_{DA} = \frac{p_A V_A - p_D V_D}{1-\gamma} \quad [\because BC \text{ is adiabatic process}]$$

$\because$ *B* and *C* lies on adiabatic curve *BC.*

$\therefore \qquad p_B V_B^{\gamma} = p_C V_C^{\gamma}$

$$p_C = p_B\left(\frac{V_B}{V_C}\right)^{\gamma} = p_B\left(\frac{1}{2}\right)^{\gamma} = 2^{-\gamma}\, p_B$$

Similarly, $\qquad p_D = 2^{-\gamma}\, p_A$

Total work done by the engine in one cycle *ABCDA*.

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = W_{BC} + W_{DA}$$

$$= \frac{(p_C V_C - p_B V_B)}{1-\gamma} + \frac{(p_A V_A - p_D V_D)}{1-\gamma}$$

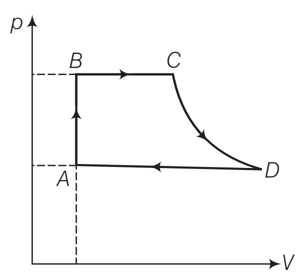
$$W = \frac{1}{1-\gamma}[2^{-\gamma} p_B (2V_B) - p_B V_B + p_A V_A - 2^{-\gamma} p_B (2V_B)]$$

$$= \frac{1}{1-\gamma}[p_B V_B (2^{-\gamma+1} - 1) - p_A V_A (2^{-\gamma+1} - 1)$$

$$= \frac{1}{1-\gamma}(2^{1-\gamma} - 1)(p_B - p_A)V_A$$

$$= \frac{3}{2}\left[1 - \left(\frac{1}{2}\right)^{2/3}\right](p_B - p_A)V_A$$

Q. 25 A cycle followed by an engine (made of one mole of an ideal gas in a cylinder with a piston) is shown in figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle. $[C_V = (3/2)R]$



(a) *AB* : constant volume

(b) *BC* : constant pressure

(c) *CD* : adiabatic

(d) *DA* : constant pressure

Thinking Process

Find amount of heat associated with each process by using first law of thermodynamics.

Ans. (a) For process *AB*,

Volume is constant, hence work done $dW = 0$

Now, by first law of thermodynamics,

$$dQ = dU + dW = dU + 0 = dU$$

$$= n\,Cv\,dT = n\,Cv\,(T_B - T_A)$$

$$= \frac{3}{2}R\,(T_B - T_A) \qquad (\because n = 1)$$

$$= \frac{3}{2}(RT_B - RT_A) = \frac{3}{2}(p_B V_B - p_A V_A)$$

Heat exchanged $= \frac{3}{2}(p_B V_B - p_A V_A)$

(b) For process BC, p = constant

$$dQ = dU + dW = \frac{3}{2}R(T_C - T_B) + p_B(V_C - V_B)$$

$$= \frac{3}{2}(p_C V_C - p_B V_B) + p_B(V_C - V_B) = \frac{5}{2}p_B(V_C - V_B)$$

Heat exchanged $= \frac{5}{2}p_B(V_C - V_A)$ $(\because p_B = p_C \text{ and } p_B = V_A)$

(c) For process CD, Because CD is adiabatic, dQ = Heat exchanged = 0

(d) DA involves compression of gas from V_D to V_A at constant pressure p_A.

$\therefore$ Heat exchanged can be calculated by similar way as BC_1,

Hence, $dQ = \frac{5}{2}p_A(V_A - V_D)$.

Q. 26 **Consider that an ideal gas (n moles) is expanding in a process given by $p = f(V)$, which passes through a point (V_0, p_0). Show that the gas is absorbing heat at (p_0, V_0) if the slope of the curve $p = f(V)$ is larger than the slope of the adiabatic passing through (p_0, V_0).**

Ans. According to question, slope of the curve $= f(V)$, where V is volume.

$\therefore$ Slope of $p = f(V)$, curve at $(V_0, p_0) = f(V_0)$

Slope of adiabatic at $(V_0, p_0) = k(-\gamma)V_0^{-1-\gamma} = -\gamma\, p_0/V_0$

Now heat absorbed in the process $p = f(V)$

$$dQ = dU + dW = nC_V dT + pdV \quad \ldots\text{(i)}$$

$\because$ $$pV = nRT \Rightarrow T = \left(\frac{1}{nR}\right)pV$$

$\Rightarrow$ $$T = \left(\frac{1}{nR}\right)V f(V)$$

$\Rightarrow$ $$dT = \left(\frac{1}{nR}\right)[f(V) + Vf'(V)]dV \quad \ldots\text{(ii)}$$

Now from Eq. (i) $$\frac{dQ}{dV} = nC_V\frac{dT}{dV} + p\frac{dV}{dV} = nC_V\frac{dT}{dV} + p$$

$$= \frac{nC_V}{nR} \times [f(V) + V f'(V)] + p \quad \text{[from Eq. (ii)]}$$

$$= \frac{C_V}{R}[f(V) + V f'(V)] + f(V) \quad [\because p = f(V)]$$

$\Rightarrow$ $$\left[\frac{dQ}{dV}\right]_{V=V_0} = \frac{C_V}{R}[f(V_0) + V_0 f'](V_0)] + f(V_0)$$

$$= f(V_0)\left[\frac{C_V}{R} + 1\right] + V_0 f'(V_0)\frac{C_V}{R}$$

$\because$ $$C_V = \frac{R}{\gamma - 1} \Rightarrow \frac{C_V}{R} = \frac{1}{\gamma - 1}$$

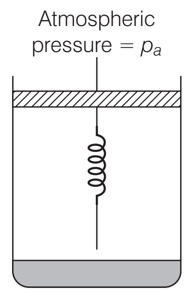
$\Rightarrow$ $$\left[\frac{dQ}{dv}\right]_{V=V_0} = \left[\frac{1}{\gamma-1} + 1\right]f(V_0) + \frac{V_0 f'(V_0)}{\gamma - 1}$$

$$= \frac{\gamma}{\gamma-1}p_0 + \frac{V_0}{\gamma-1}f'(V_0)$$

Heat is absorbed where $\frac{dQ}{dV} > 0$, when gas expands

Hence, $\gamma\, p_0 + V_0 f'(V_0) > 0$ or $f'(V_0) > \left(-\gamma\frac{p_o}{V_o}\right)$

Q. 27 Consider one mole of perfect gas in a cylinder of unit cross-section with a piston attached (figure). A spring (spring constant k)is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of value from V_0 to V_1.



(a) What is the initial pressure of the system?

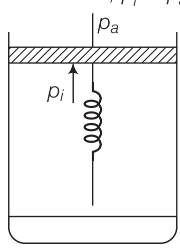
(b) What is the final pressure of the system?

(c) Using the first law of thermodynamics, write down a relation between Q, p_a, V, V_0 and k.

Thinking Process

We will assume the piston is massless, hence, at equilibrium atmospheric pressure and inside pressure will be same.

Ans. (a) Initially the piston is in equilibrium hence, $p_i = p_a$



(b) On supplying heat, the gas expands from V_0 to V_1

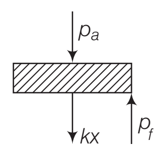
$\therefore$ Increase in volume of the gas $= V_1 - V_0$

As the piston is of unit cross-sectional area hence, extension in the spring

$$x = \frac{V_1 - V_0}{\text{Area}} = V_1 - V_0 \qquad [\text{Area} = 1]$$

$\therefore$ Force exerted by the spring on the piston

$$= F = kx = k(V_1 - V_0)$$



Hence, final pressure $= p_f = p_a + kx$

$$= p_a + k \times (V_1 - V_0)$$

(c) From first law of thermodynamics $dQ = dU + dW$

If T is final temperature of the gas, then increase in internal energy

$$dU = C_V (T, - T_o) = C_V (T, - T_0)$$

We can write, $$T = \frac{p_f V_1}{R} = \left[\frac{p_a + k(V_1 - V_0)}{R}\right]\frac{V_1}{R}$$

Work done by the gas $= pdV +$ increase in PE of the spring

$$= p_a (V_1 - V_0) + \frac{1}{2} kx^2$$

Now, we can write $dQ = dU + dW$

$$= C_V (T - T_0) + p_a (V - V_0) + \frac{1}{2} kx^2$$

$$= C_V (T - T_0) + p_a (V - V_0) + \frac{1}{2} (V_1 - V_0)^2$$

This is the required relation.