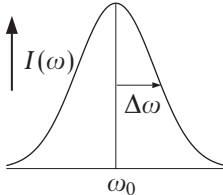


8.8 Line radiation

Spectral line broadening

Natural broadening ^a	$I(\omega) = \frac{(2\pi\tau)^{-1}}{(2\tau)^{-2} + (\omega - \omega_0)^2} \quad (8.112)$	$I(\omega)$ normalised intensity ^b τ lifetime of excited state ω angular frequency ($= 2\pi\nu$)
Natural half-width	$\Delta\omega = \frac{1}{2\tau} \quad (8.113)$	$\Delta\omega$ half-width at half-power ω_0 centre frequency
Collision broadening	$I(\omega) = \frac{(\pi\tau_c)^{-1}}{(\tau_c)^{-2} + (\omega - \omega_0)^2} \quad (8.114)$	τ_c mean time between collisions p pressure d effective atomic diameter m gas particle mass k Boltzmann constant T temperature c speed of light
Collision and pressure half-width ^c	$\Delta\omega = \frac{1}{\tau_c} = p\pi d^2 \left(\frac{\pi mkT}{16} \right)^{-1/2} \quad (8.115)$	
Doppler broadening	$I(\omega) = \left(\frac{mc^2}{2kT\omega_0^2\pi} \right)^{1/2} \exp \left[-\frac{mc^2}{2kT} \frac{(\omega - \omega_0)^2}{\omega_0^2} \right] \quad (8.116)$	
Doppler half-width	$\Delta\omega = \omega_0 \left(\frac{2kT \ln 2}{mc^2} \right)^{1/2} \quad (8.117)$	

^aThe transition probability per unit time for the state is $= 1/\tau$. In the classical limit of a damped oscillator, the e-folding time of the electric field is 2τ . Both the natural and collision profiles described here are Lorentzian.

^bThe intensity spectra are normalised so that $\int I(\omega) d\omega = 1$, assuming $\Delta\omega/\omega_0 \ll 1$.

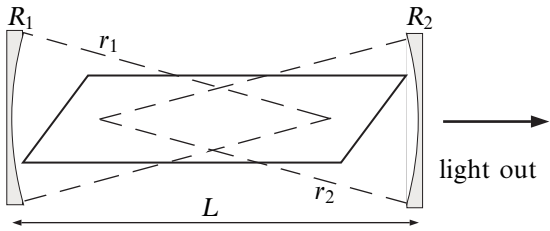
^cThe pressure-broadening relation combines Equations (5.78), (5.86) and (5.89) and assumes an otherwise perfect gas of finite-sized atoms. More accurate expressions are considerably more complicated.

Einstein coefficients^a

Absorption	$R_{12} = B_{12}I_\nu n_1 \quad (8.118)$	R_{ij} transition rate, level $i \rightarrow j$ ($\text{m}^{-3} \text{s}^{-1}$) B_{ij} Einstein B coefficients I_ν specific intensity of radiation field
Spontaneous emission	$R_{21} = A_{21}n_2 \quad (8.119)$	A_{21} Einstein A coefficient n_i number density of atoms in quantum level i (m^{-3})
Stimulated emission	$R'_{21} = B_{21}I_\nu n_2 \quad (8.120)$	
Coefficient ratios	$\frac{A_{21}}{B_{12}} = \frac{2h\nu^3}{c^2} \frac{g_1}{g_2} \quad (8.121)$	h Planck constant ν frequency c speed of light
	$\frac{B_{21}}{B_{12}} = \frac{g_1}{g_2} \quad (8.122)$	g_i degeneracy of i th level

^aNote that the coefficients can also be defined in terms of spectral energy density, $u_\nu = 4\pi I_\nu/c$ rather than I_ν . In this case $\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3} \frac{g_1}{g_2}$. See also *Population densities* on page 116.

Lasers^a

		
Cavity stability condition	$0 \leq \left(1 - \frac{L}{r_1}\right) \left(1 - \frac{L}{r_2}\right) \leq 1$ (8.123)	$r_{1,2}$ radii of curvature of end-mirrors L distance between mirror centres
Longitudinal cavity modes ^b	$v_n = \frac{c}{2L} n$ (8.124)	v_n mode frequency n integer c speed of light
Cavity Q	$Q = \frac{2\pi L (R_1 R_2)^{1/4}}{\lambda [1 - (R_1 R_2)^{1/2}]}$ (8.125)	Q quality factor $R_{1,2}$ mirror (power) reflectances λ wavelength
	$\simeq \frac{4\pi L}{\lambda (1 - R_1 R_2)}$ (8.126)	
Cavity line width	$\Delta v_c = \frac{v_n}{Q} = 1/(2\pi\tau_c)$ (8.127)	Δv_c cavity line width (FWHP) τ_c cavity photon lifetime
Schawlow–Townes line width	$\frac{\Delta v}{v_n} = \frac{2\pi h (\Delta v_c)^2}{P} \left(\frac{g_l N_u}{g_l N_u - g_u N_l} \right)$ (8.128)	Δv line width (FWHP) P laser power $g_{u,l}$ degeneracy of upper/lower levels $N_{u,l}$ number density of upper/lower levels
Threshold lasing condition	$R_1 R_2 \exp[2(\alpha - \beta)L] > 1$ (8.129)	α gain per unit length of medium β loss per unit length of medium

^aAlso see the *Fabry-Perot etalon* on page 163. Note that “cavity” refers to the empty cavity, with no lasing medium present.

^bThe mode spacing equals the cavity free spectral range.