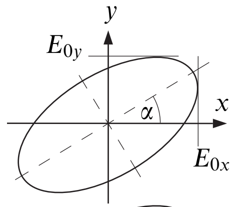
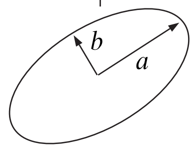
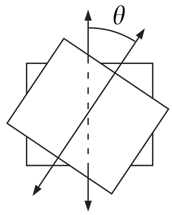
8.6 Polarisation

Elliptical polarisation[a]

Elliptical polarisation	$\boldsymbol{E}=(E_{0x},E_{0y}e^{\mathbf{i}\delta})e^{\mathbf{i}(kz-\omega t)}$	(8.80)	$\boldsymbol{E}$ k z ωt	electric field wavevector propagation axis angular frequency × time
Polarisation angle[b]	$\tan 2\alpha=\dfrac{2E_{0x}E_{0y}}{E_{0x}^2-E_{0y}^2}\cos\delta$	(8.81)	E_{0x} E_{0y} δ	x amplitude of $\boldsymbol{E}$ y amplitude of $\boldsymbol{E}$ relative phase of E_y with respect to E_x
Ellipticity[c]	$e=\dfrac{a-b}{a}$	(8.82)	α e a b	polarisation angle ellipticity semi-major axis semi-minor axis
Malus's law[d]	$I(\theta)=I_0\cos^2\theta$	(8.83)	$I(\theta)$ I_0 θ	transmitted intensity incident intensity polariser–analyser angle







[a]See the introduction (page 161) for a discussion of sign and handedness conventions.
[b]Angle between ellipse major axis and x axis. Sometimes the polarisation angle is defined as $\pi/2-\alpha$.
[c]This is one of several definitions for ellipticity.
[d]Transmission through skewed polarisers for unpolarised incident light.

Jones vectors and matrices

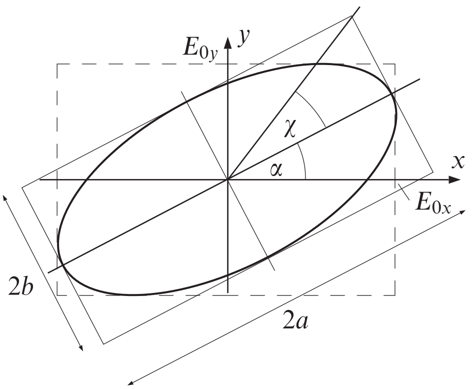
Normalised electric field[a]	$\boldsymbol{E}=\begin{pmatrix}E_x\\E_y\end{pmatrix};\quad \lvert\boldsymbol{E}\rvert=1$	(8.84)	$\boldsymbol{E}$ E_x E_y	electric field x component of $\boldsymbol{E}$ y component of $\boldsymbol{E}$
Example vectors:	$E_x=\begin{pmatrix}1\\0\end{pmatrix}$ $\quad E_{45}=\dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $E_{\mathrm{r}}=\dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\-\mathbf{i}\end{pmatrix}$ $\quad E_{\mathrm{l}}=\dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\\mathbf{i}\end{pmatrix}$		E_{45} E_{r} E_{l}	45° to x axis right-hand circular left-hand circular
Jones matrix	$\boldsymbol{E}_{\mathrm{t}}=\mathbf{A}\boldsymbol{E}_{\mathrm{i}}$	(8.85)	$\boldsymbol{E}_{\mathrm{t}}$ $\boldsymbol{E}_{\mathrm{i}}$ $\mathbf{A}$	transmitted vector incident vector Jones matrix

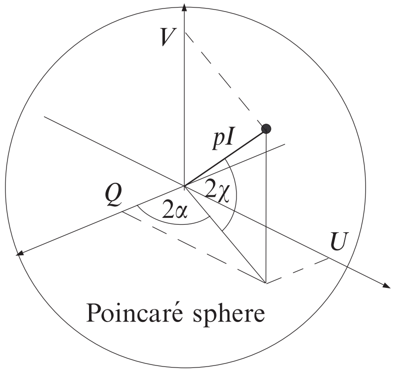
Example matrices:

Linear polariser $\parallel x$	$\begin{pmatrix}1&0\\0&0\end{pmatrix}$	Linear polariser $\parallel y$	$\begin{pmatrix}0&0\\0&1\end{pmatrix}$
Linear polariser at 45°	$\dfrac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	Linear polariser at −45°	$\dfrac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$
Right circular polariser	$\dfrac{1}{2}\begin{pmatrix}1&\mathbf{i}\\-\mathbf{i}&1\end{pmatrix}$	Left circular polariser	$\dfrac{1}{2}\begin{pmatrix}1&-\mathbf{i}\\\mathbf{i}&1\end{pmatrix}$
$\lambda/4$ plate (fast $\parallel x$)	$e^{\mathbf{i}\pi/4}\begin{pmatrix}1&0\\0&\mathbf{i}\end{pmatrix}$	$\lambda/4$ plate (fast $\perp x$)	$e^{\mathbf{i}\pi/4}\begin{pmatrix}1&0\\0&-\mathbf{i}\end{pmatrix}$

[a]Known as the "normalised Jones vector."

Stokes parameters[a]





Electric fields	$E_x = E_{0x}\mathrm{e}^{\mathbf{i}(kz-\omega t)}$	(8.86)	k wavevector
	$E_y = E_{0y}\mathrm{e}^{\mathbf{i}(kz-\omega t+\delta)}$	(8.87)	ωt angular frequency × time; δ relative phase of E_y with respect to E_x
Axial ratio[b]	$\tan\chi = \pm r = \pm\frac{b}{a}$	(8.88)	χ (see diagram); r axial ratio
Stokes parameters	$I = \langle E_x^2\rangle + \langle E_y^2\rangle$	(8.89)	
	$Q = \langle E_x^2\rangle - \langle E_y^2\rangle$	(8.90)	E_x electric field component $\parallel x$
	$= pI\cos 2\chi \cos 2\alpha$	(8.91)	E_y electric field component $\parallel y$
	$U = 2\langle E_x E_y\rangle \cos\delta$	(8.92)	E_{0x} field amplitude in x direction
	$= pI\cos 2\chi \sin 2\alpha$	(8.93)	E_{0y} field amplitude in y direction
	$V = 2\langle E_x E_y\rangle \sin\delta$	(8.94)	α polarisation angle; p degree of polarisation
	$= pI\sin 2\chi$	(8.95)	$\langle\cdot\rangle$ mean over time
Degree of polarisation	$p = \frac{(Q^2+U^2+V^2)^{1/2}}{I} \leq 1$	(8.96)	

	Q/I	U/I	V/I		Q/I	U/I	V/I
left circular	0	0	−1	right circular	0	0	1
linear $\parallel x$	1	0	0	linear $\parallel y$	−1	0	0
linear 45° to x	0	1	0	linear −45° to x	0	−1	0
unpolarised	0	0	0				

[a]Using the convention that right-handed circular polarisation corresponds to a clockwise rotation of the electric field in a given plane when looking towards the source. The propagation direction in the diagram is out of the plane. The parameters I, Q, U, and V are sometimes denoted s_0, s_1, s_2, and s_3, and other nomenclatures exist. There is no generally accepted definition – often the parameters are scaled to be dimensionless, with $s_0 = 1$, or to represent power flux through a plane ⊥ the beam, i.e., $I = (\langle E_x^2\rangle + \langle E_y^2\rangle)/Z_0$ etc., where Z_0 is the impedance of free space.

[b]The axial ratio is positive for right-handed polarisation and negative for left-handed polarisation using our definitions.