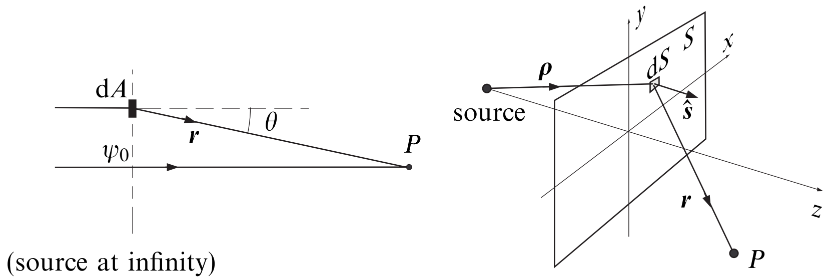
8.4 Fresnel diffraction

Kirchhoff's diffraction formula[a]

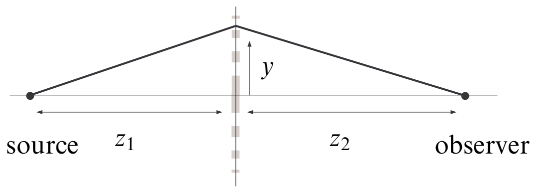


Source at infinity	$\psi_P = -\frac{\mathrm{i}}{\lambda}\psi_0 \int_{\text{plane}} K(\theta)\frac{\mathrm{e}^{\mathrm{i}kr}}{r}\,\mathrm{d}A$	(8.45)
where:		
Obliquity factor (cardioid)	$K(\theta) = \frac{1}{2}(1+\cos\theta)$	(8.46)
Source at finite distance[b]	$\psi_P = -\frac{\mathrm{i}E_0}{\lambda} \oint_{\text{closed surface}} \frac{\mathrm{e}^{\mathrm{i}k(\rho+r)}}{2\rho r}[\cos(\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{r}}) - \cos(\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{\rho}})]\,\mathrm{d}S$	(8.47)

ψ_P	complex amplitude at P
λ	wavelength
k	wavenumber $(=2\pi/\lambda)$
ψ_0	incident amplitude
θ	obliquity angle
r	distance of $\mathrm{d}A$ from P $(\gg \lambda)$
$\mathrm{d}A$	area element on incident wavefront
K	obliquity factor
$\mathrm{d}S$	element of closed surface
$\hat{}$	unit vector
$\boldsymbol{s}$	vector normal to $\mathrm{d}S$
$\boldsymbol{r}$	vector from P to $\mathrm{d}S$
$\boldsymbol{\rho}$	vector from source to $\mathrm{d}S$
E_0	amplitude (see footnote)

[a] Also known as the "Fresnel–Kirchhoff formula." Diffraction by an obstacle coincident with the integration surface can be approximated by omitting that part of the surface from the integral.
[b] The source amplitude at $\boldsymbol{\rho}$ is $\psi(\rho) = E_0\mathrm{e}^{\mathrm{i}k\rho}/\rho$. The integral is taken over a surface enclosing the point P.

Fresnel zones

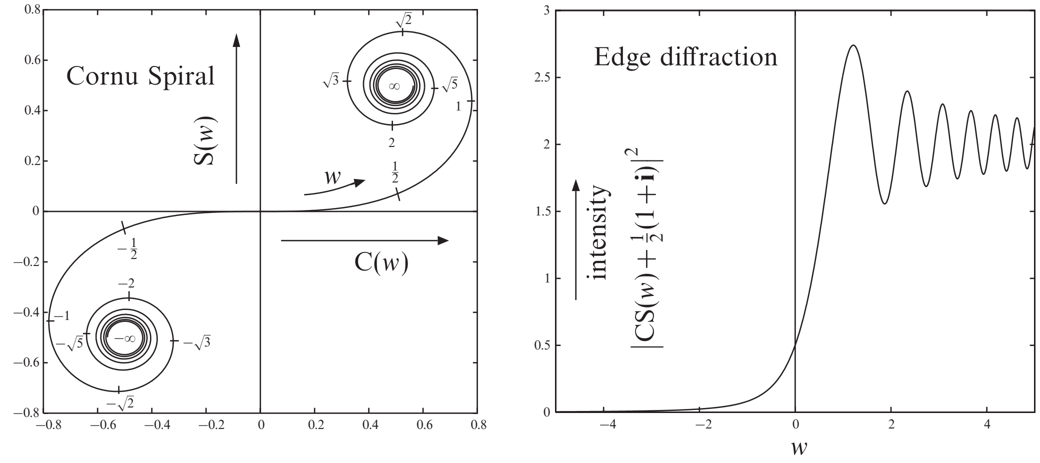


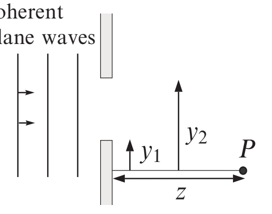
Effective aperture distance[a]	$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$	(8.48)
Half-period zone radius	$y_n = (n\lambda z)^{1/2}$	(8.49)
Axial zeros (circular aperture)	$z_m = \frac{R^2}{2m\lambda}$	(8.50)

z	effective distance
z_1	source–aperture distance
z_2	aperture–observer distance
n	half-period zone number
λ	wavelength
y_n	nth half-period zone radius
z_m	distance of mth zero from aperture
R	aperture radius

[a] I.e., the aperture–observer distance to be employed when the source is not at infinity.

Cornu spiral



Fresnel integrals[a]	$$\mathrm{C}(w)=\int_0^w \cos\frac{\pi t^2}{2}\,\mathrm{d}t \qquad (8.51)$$ $$\mathrm{S}(w)=\int_0^w \sin\frac{\pi t^2}{2}\,\mathrm{d}t \qquad (8.52)$$	C Fresnel cosine integral S Fresnel sine integral
Cornu spiral	$$\mathrm{CS}(w)=\mathrm{C}(w)+\mathbf{i}\mathrm{S}(w) \qquad (8.53)$$ $$\mathrm{CS}(\pm\infty)=\pm\frac{1}{2}(1+\mathbf{i}) \qquad (8.54)$$	CS Cornu spiral v,w length along spiral
Edge diffraction	$$\psi_P=\frac{\psi_0}{2^{1/2}}\left[\mathrm{CS}(w)+\frac{1}{2}(1+\mathbf{i})\right] \qquad (8.55)$$ $$\text{where} \quad w=y\left(\frac{2}{\lambda z}\right)^{1/2} \qquad (8.56)$$	ψ_P complex amplitude at P ψ_0 unobstructed amplitude λ wavelength z distance of P from aperture plane [see (8.48)] y position of edge
Diffraction from a long slit[b]	$$\psi_P=\frac{\psi_0}{2^{1/2}}[\mathrm{CS}(w_2)-\mathrm{CS}(w_1)] \qquad (8.57)$$ $$\text{where} \quad w_i=y_i\left(\frac{2}{\lambda z}\right)^{1/2} \qquad (8.58)$$	coherent plane waves 
Diffraction from a rectangular aperture	$$\psi_P=\frac{\psi_0}{2}[\mathrm{CS}(v_2)-\mathrm{CS}(v_1)]\times \qquad (8.59)$$ $$[\mathrm{CS}(w_2)-\mathrm{CS}(w_1)] \qquad (8.60)$$ $$\text{where} \quad v_i=x_i\left(\frac{2}{\lambda z}\right)^{1/2} \qquad (8.61)$$ $$\text{and} \quad w_i=y_i\left(\frac{2}{\lambda z}\right)^{1/2} \qquad (8.62)$$	x_i positions of slit sides y_i positions of slit top/bottom

[a]See also Equation (2.393) on page 45.
[b]Slit long in x.