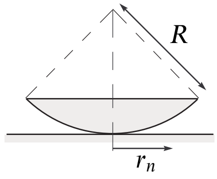
8.2 Interference

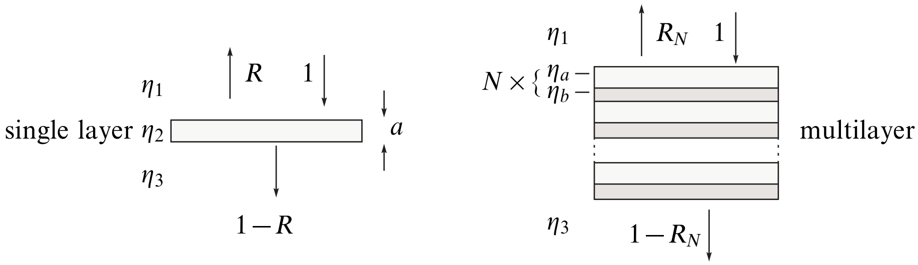
Newton's rings[a]

nth dark ring	$r_n^2 = nR\lambda_0$	(8.1)	r_n radius of nth ring n integer (≥ 0) R lens radius of curvature
nth bright ring	$r_n^2 = \left(n+\frac{1}{2}\right)R\lambda_0$	(8.2)	λ_0 wavelength in external medium



[a]Viewed in reflection.

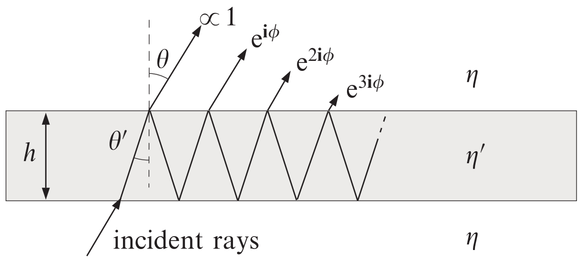
Dielectric layers[a]



Quarter-wave condition	$a = \frac{m}{\eta_2}\frac{\lambda_0}{4}$	(8.3)	a film thickness m thickness integer ($m \geq 0$) η_2 film refractive index λ_0 free-space wavelength R power reflectance coefficient η_1 entry-side refractive index η_3 exit-side refractive index
Single-layer reflectance[b]	$R = \begin{cases} \left(\frac{\eta_1\eta_3 - \eta_2^2}{\eta_1\eta_3 + \eta_2^2}\right)^2 & (m \text{ odd}) \\ \left(\frac{\eta_1 - \eta_3}{\eta_1 + \eta_3}\right)^2 & (m \text{ even}) \end{cases}$	(8.4)	
Dependence of R on layer thickness, m	max if $(-1)^m(\eta_1-\eta_2)(\eta_2-\eta_3) > 0$	(8.5)	
	min if $(-1)^m(\eta_1-\eta_2)(\eta_2-\eta_3) < 0$	(8.6)	
	$R = 0$ if $\eta_2 = (\eta_1\eta_3)^{1/2}$ and m odd	(8.7)	
Multilayer reflectance[c]	$R_N = \left[\frac{\eta_1 - \eta_3(\eta_a/\eta_b)^{2N}}{\eta_1 + \eta_3(\eta_a/\eta_b)^{2N}}\right]^2$	(8.8)	R_N multilayer reflectance N number of layer pairs η_a refractive index of top layer η_b refractive index of bottom layer

[a]For normal incidence, assuming the quarter-wave condition. The media are also assumed lossless, with $\mu_r = 1$.
[b]See page 154 for the definition of R.
[c]For a stack of N layer pairs, giving an overall refractive index sequence $\eta_1\eta_a, \eta_b\eta_a \ldots \eta_a\eta_b\eta_3$ (see right-hand diagram). Each layer in the stack meets the quarter-wave condition with $m = 1$.

Fabry-Perot etalon[a]



Incremental phase difference[b]	$\phi = 2k_0 h\eta' \cos\theta'$	(8.9)	ϕ incremental phase difference k_0 free-space wavenumber ($=2\pi/\lambda_0$) h cavity width θ fringe inclination (usually $\ll 1$) θ' internal angle of refraction η' cavity refractive index η external refractive index n fringe order (integer)
	$= 2k_0 h\eta' \left[1 - \left(\frac{\eta \sin\theta}{\eta'}\right)^2\right]^{1/2}$	(8.10)	
	$= 2\pi n$ for a maximum	(8.11)	
Coefficient of finesse	$F = \frac{4R}{(1-R)^2}$	(8.12)	F coefficient of finesse R interface power reflectance
Finesse	$\mathscr{F} = \frac{\pi}{2} F^{1/2}$	(8.13)	$\mathscr{F}$ finesse λ_0 free-space wavelength Q cavity quality factor
	$= \frac{\lambda_0}{\eta' h} Q$	(8.14)	
Transmitted intensity	$I(\theta) = \frac{I_0 (1-R)^2}{1 + R^2 - 2R\cos\phi}$	(8.15)	I transmitted intensity I_0 incident intensity A Airy function
	$= \frac{I_0}{1 + F \sin^2(\phi/2)}$	(8.16)	
	$= I_0 A(\theta)$	(8.17)	
Fringe intensity profile	$\Delta\phi = 2 \arcsin(F^{-1/2})$	(8.18)	$\Delta\phi$ phase difference at half intensity point
	$\simeq 2F^{-1/2}$	(8.19)	
Chromatic resolving power	$\frac{\lambda_0}{\delta\lambda} \simeq \frac{R^{1/2}\pi n}{1-R} = n\mathscr{F}$	(8.20)	$\delta\lambda$ minimum resolvable wavelength difference
	$\simeq \frac{2\mathscr{F} h\eta'}{\lambda_0}$ $(\theta \ll 1)$	(8.21)	
Free spectral range[c]	$\delta\lambda_f = \mathscr{F}\,\delta\lambda$	(8.22)	$\delta\lambda_f$ wavelength free spectral range $\delta\nu_f$ frequency free spectral range
	$\delta\nu_f = \frac{c}{2\eta' h}$	(8.23)	

[a] Neglecting any effects due to surface coatings on the etalon. See also *Lasers* on page 174.
[b] Between adjacent rays. Highest order fringes are near the centre of the pattern.
[c] At near-normal incidence ($\theta \simeq 0$), the orders of two spectral components separated by $< \delta\lambda_f$ will not overlap.