7.7 Transmission lines and waveguides

Transmission line relations

Loss-free transmission line equations	$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \quad (7.170)$$ $$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} \quad (7.171)$$	V potential difference across line; I current in line; L inductance per unit length; C capacitance per unit length
Wave equation for a lossless transmission line	$$\frac{1}{LC}\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2} \quad (7.172)$$ $$\frac{1}{LC}\frac{\partial^2 I}{\partial x^2} = \frac{\partial^2 I}{\partial t^2} \quad (7.173)$$	x distance along line; t time
Characteristic impedance of lossless line	$$Z_{\mathrm{c}} = \sqrt{\frac{L}{C}} \quad (7.174)$$	Z_{c} characteristic impedance
Characteristic impedance of lossy line	$$\boldsymbol{Z}_{\mathrm{c}} = \sqrt{\frac{R+\mathrm{i}\omega L}{G+\mathrm{i}\omega C}} \quad (7.175)$$	R resistance per unit length of conductor; G conductance per unit length of insulator; ω angular frequency
Wave speed along a lossless line	$$v_{\mathrm{p}} = v_{\mathrm{g}} = \frac{1}{\sqrt{LC}} \quad (7.176)$$	v_{p} phase speed; v_{g} group speed
Input impedance of a terminated lossless line	$$\boldsymbol{Z}_{\mathrm{in}} = Z_{\mathrm{c}}\frac{\boldsymbol{Z}_{\mathrm{t}}\cos kl - \mathrm{i}Z_{\mathrm{c}}\sin kl}{Z_{\mathrm{c}}\cos kl - \mathrm{i}\boldsymbol{Z}_{\mathrm{t}}\sin kl} \quad (7.177)$$ $$= Z_{\mathrm{c}}^2/\boldsymbol{Z}_{\mathrm{t}} \quad \text{if } l=\lambda/4 \quad (7.178)$$	$\boldsymbol{Z}_{\mathrm{in}}$ (complex) input impedance; $\boldsymbol{Z}_{\mathrm{t}}$ (complex) terminating impedance; k wavenumber ($=2\pi/\lambda$)
Reflection coefficient from a terminated line	$$\boldsymbol{r} = \frac{\boldsymbol{Z}_{\mathrm{t}}-\boldsymbol{Z}_{\mathrm{c}}}{\boldsymbol{Z}_{\mathrm{t}}+\boldsymbol{Z}_{\mathrm{c}}} \quad (7.179)$$	l distance from termination; $\boldsymbol{r}$ (complex) voltage reflection coefficient
Line voltage standing wave ratio	$$\mathrm{VSWR} = \frac{1+\lvert\boldsymbol{r}\rvert}{1-\lvert\boldsymbol{r}\rvert} \quad (7.180)$$	

Transmission line impedances[a]

Coaxial line	$$Z_{\mathrm{c}} = \sqrt{\frac{\mu}{4\pi^2\epsilon}}\ln\frac{b}{a} \simeq \frac{60}{\sqrt{\epsilon_{\mathrm{r}}}}\ln\frac{b}{a} \quad (7.181)$$	Z_{c} characteristic impedance (Ω); a radius of inner conductor; b radius of outer conductor; ϵ permittivity ($=\epsilon_0\epsilon_{\mathrm{r}}$)
Open wire feeder	$$Z_{\mathrm{c}} = \sqrt{\frac{\mu}{\pi^2\epsilon}}\ln\frac{l}{r} \simeq \frac{120}{\sqrt{\epsilon_{\mathrm{r}}}}\ln\frac{l}{r} \quad (7.182)$$	μ permeability ($=\mu_0\mu_{\mathrm{r}}$); r radius of wires; l distance between wires ($\gg r$)
Paired strip	$$Z_{\mathrm{c}} = \sqrt{\frac{\mu}{\epsilon}}\frac{d}{w} \simeq \frac{377}{\sqrt{\epsilon_{\mathrm{r}}}}\frac{d}{w} \quad (7.183)$$	d strip separation; w strip width ($\gg d$)
Microstrip line	$$Z_{\mathrm{c}} \simeq \frac{377}{\sqrt{\epsilon_{\mathrm{r}}}[(w/h)+2]} \quad (7.184)$$	h height above earth plane ($\ll w$)

[a] For lossless lines.

Waveguides[a]

Waveguide equation	$k_{\mathrm{g}}^2 = \frac{\omega^2}{c^2} - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}$	(7.185)	k_{g} wavenumber in guide ω angular frequency a guide height b guide width m,n mode indices with respect to a and b (integers) c speed of light
Guide cutoff frequency	$\nu_{\mathrm{c}} = c\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}$	(7.186)	ν_{c} cutoff frequency ω_{c} $2\pi\nu_{\mathrm{c}}$
Phase velocity above cutoff	$v_{\mathrm{p}} = \frac{c}{\sqrt{1-(\nu_{\mathrm{c}}/\nu)^2}}$	(7.187)	v_{p} phase velocity ν frequency
Group velocity above cutoff	$v_{\mathrm{g}} = c^2/v_{\mathrm{p}} = c\sqrt{1-(\nu_{\mathrm{c}}/\nu)^2}$	(7.188)	v_{g} group velocity
Wave impedances[b]	$Z_{\mathrm{TM}} = Z_0\sqrt{1-(\nu_{\mathrm{c}}/\nu)^2}$ $Z_{\mathrm{TE}} = Z_0/\sqrt{1-(\nu_{\mathrm{c}}/\nu)^2}$	(7.189) (7.190)	Z_{TM} wave impedance for transverse magnetic modes Z_{TE} wave impedance for transverse electric modes Z_0 impedance of free space $(=\sqrt{\mu_0/\epsilon_0})$

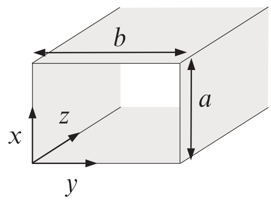
Field solutions for TE_{mn} modes[c]

$$B_x = \frac{\mathbf{i}k_{\mathrm{g}}c^2}{\omega_{\mathrm{c}}^2}\frac{\partial B_z}{\partial x} \qquad E_x = \frac{\mathbf{i}\omega c^2}{\omega_{\mathrm{c}}^2}\frac{\partial B_z}{\partial y}$$

$$B_y = \frac{\mathbf{i}k_{\mathrm{g}}c^2}{\omega_{\mathrm{c}}^2}\frac{\partial B_z}{\partial y} \qquad E_y = \frac{-\mathbf{i}\omega c^2}{\omega_{\mathrm{c}}^2}\frac{\partial B_z}{\partial x} \qquad (7.191)$$

$$B_z = B_0 \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \qquad E_z = 0$$

Field solutions for TM_{mn} modes[c]



$$E_x = \frac{\mathbf{i}k_{\mathrm{g}}c^2}{\omega_{\mathrm{c}}^2}\frac{\partial E_z}{\partial x} \qquad B_x = \frac{-\mathbf{i}\omega}{\omega_c^2}\frac{\partial E_z}{\partial y}$$

$$E_y = \frac{\mathbf{i}k_{\mathrm{g}}c^2}{\omega_{\mathrm{c}}^2}\frac{\partial E_z}{\partial y} \qquad B_y = \frac{\mathbf{i}\omega}{\omega_c^2}\frac{\partial E_z}{\partial x} \qquad (7.192)$$

$$E_z = E_0 \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad B_z = 0$$

[a]Equations are for lossless waveguides with rectangular cross sections and no dielectric.
[b]The ratio of the electric field to the magnetic field strength in the xy plane.
[c]Both TE and TM modes propagate in the z direction with a further factor of $\exp[\mathbf{i}(k_{\mathrm{g}}z-\omega t)]$ on all components. B_0 and E_0 are the amplitudes of the z components of magnetic flux density and electric field respectively.