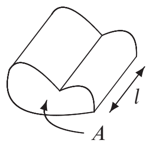
7.6 LCR circuits

LCR definitions

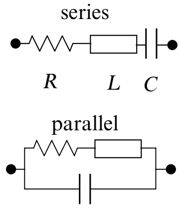
Current	$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$	(7.139)	I current Q charge
Ohm's law	$V = IR$	(7.140)	R resistance V potential difference over R I current through R
Ohm's law (field form)	$\boldsymbol{J} = \sigma \boldsymbol{E}$	(7.141)	$\boldsymbol{J}$ current density $\boldsymbol{E}$ electric field σ conductivity
Resistivity	$\rho = \frac{1}{\sigma} = \frac{RA}{l}$	(7.142)	ρ resistivity A area of face (I is normal to face) l length
Capacitance	$C = \frac{Q}{V}$	(7.143)	C capacitance V potential difference across C
Current through capacitor	$I = C\frac{\mathrm{d}V}{\mathrm{d}t}$	(7.144)	I current through C t time
Self-inductance	$L = \frac{\Phi}{I}$	(7.145)	Φ total linked flux I current through inductor
Voltage across inductor	$V = -L\frac{\mathrm{d}I}{\mathrm{d}t}$	(7.146)	V potential difference over L
Mutual inductance	$L_{12} = \frac{\Phi_1}{I_2} = L_{21}$	(7.147)	Φ_1 total flux from loop 2 linked by loop 1 L_{12} mutual inductance I_2 current through loop 2
Coefficient of coupling	$\lvert L_{12} \rvert = k\sqrt{L_1 L_2}$	(7.148)	k coupling coefficient between L_1 and L_2 (≤ 1)
Linked magnetic flux through a coil	$\Phi = N\phi$	(7.149)	Φ linked flux N number of turns around ϕ ϕ flux through area of turns



Resonant LCR circuits

Phase resonant frequency[a]	$\omega_0^2 = \begin{cases} 1/LC & \text{(series)} \\ 1/LC - R^2/L^2 & \text{(parallel)} \end{cases}$	(7.150)
Tuning[b]	$\dfrac{\delta\omega}{\omega_0} = \dfrac{1}{Q} = \dfrac{R}{\omega_0 L}$	(7.151)
Quality factor	$Q = 2\pi \dfrac{\text{stored energy}}{\text{energy lost per cycle}}$	(7.152)

ω_0	resonant angular frequency
L	inductance
C	capacitance
R	resistance
$\delta\omega$	half-power bandwidth
Q	quality factor



[a]At which the impedance is purely real.
[b]Assuming the capacitor is purely reactive. If L and R are parallel, then $1/Q = \omega_0 L/R$.

Energy in capacitors, inductors, and resistors

Energy stored in a capacitor	$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$	(7.153)
Energy stored in an inductor	$U = \frac{1}{2}LI^2 = \frac{1}{2}\Phi I = \frac{1}{2}\frac{\Phi^2}{L}$	(7.154)
Power dissipated in a resistor[a] (Joule's law)	$W = IV = I^2R = \dfrac{V^2}{R}$	(7.155)
Relaxation time	$\tau = \dfrac{\epsilon_0\epsilon_r}{\sigma}$	(7.156)

U	stored energy
C	capacitance
Q	charge
V	potential difference
L	inductance
Φ	linked magnetic flux
I	current
W	power dissipated
R	resistance
τ	relaxation time
ϵ_r	relative permittivity
σ	conductivity

[a]This is d.c., or instantaneous a.c., power.

Electrical impedance

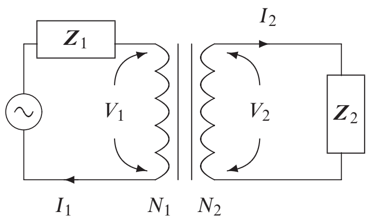
Impedances in series	$\boldsymbol{Z}_{\text{tot}} = \sum_n \boldsymbol{Z}_n$	(7.157)
Impedances in parallel	$\boldsymbol{Z}_{\text{tot}} = \left(\sum_n \boldsymbol{Z}_n^{-1}\right)^{-1}$	(7.158)
Impedance of capacitance	$\boldsymbol{Z}_{\text{C}} = -\dfrac{\mathbf{i}}{\omega C}$	(7.159)
Impedance of inductance	$\boldsymbol{Z}_{\text{L}} = \mathbf{i}\omega L$	(7.160)

Impedance: $\boldsymbol{Z}$	Capacitance: C
Inductance: L	Resistance: $R = \text{Re}[\boldsymbol{Z}]$
Conductance: $G = 1/R$	Reactance: $X = \text{Im}[\boldsymbol{Z}]$
Admittance: $\boldsymbol{Y} = 1/\boldsymbol{Z}$	Susceptance: $S = 1/X$

Kirchhoff's laws

Current law	$\sum_{\text{node}} I_i = 0$	(7.161)	I_i currents impinging on node
Voltage law	$\sum_{\text{loop}} V_i = 0$	(7.162)	V_i potential differences around loop

Transformers[a]

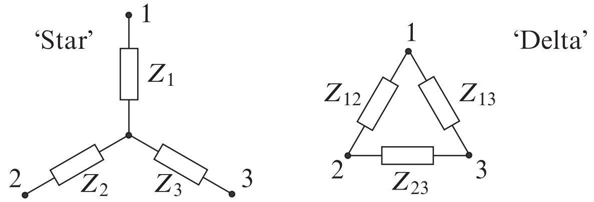


n	turns ratio
N_1	number of primary turns
N_2	number of secondary turns
V_1	primary voltage
V_2	secondary voltage
I_1	primary current
I_2	secondary current
$\boldsymbol{Z}_{\text{out}}$	output impedance
$\boldsymbol{Z}_{\text{in}}$	input impedance
$\boldsymbol{Z}_1$	source impedance
$\boldsymbol{Z}_2$	load impedance

Turns ratio	$n = N_2/N_1$	(7.163)
Transformation of voltage and current	$V_2 = nV_1$	(7.164)
	$I_2 = I_1/n$	(7.165)
Output impedance (seen by $\boldsymbol{Z}_2$)	$\boldsymbol{Z}_{\text{out}} = n^2\boldsymbol{Z}_1$	(7.166)
Input impedance (seen by $\boldsymbol{Z}_1$)	$\boldsymbol{Z}_{\text{in}} = \boldsymbol{Z}_2/n^2$	(7.167)

[a]Ideal, with a coupling constant of 1 between loss-free windings.

Star–delta transformation



i,j,k	node indices (1,2, or 3)
Z_i	impedance on node i
Z_{ij}	impedance connecting nodes i and j

Star impedances	$Z_i = \frac{Z_{ij}Z_{ik}}{Z_{ij}+Z_{ik}+Z_{jk}}$	(7.168)
Delta impedances	$Z_{ij} = Z_iZ_j\left(\frac{1}{Z_i}+\frac{1}{Z_j}+\frac{1}{Z_k}\right)$	(7.169)