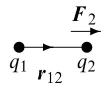
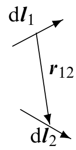
7.5 Force, torque, and energy

Electromagnetic force and torque

Force between two static charges: Coulomb's law	$\boldsymbol{F}_2=\frac{q_1q_2}{4\pi\epsilon_0 r_{12}^2}\hat{\boldsymbol{r}}_{12}$ (7.119)	$\boldsymbol{F}_2$ $q_{1,2}$ $\boldsymbol{r}_{12}$ $\hat{}$ ϵ_0	force on q_2 charges vector from 1 to 2 unit vector permittivity of free space
Force between two current-carrying elements	$\mathrm{d}\boldsymbol{F}_2=\frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[\mathrm{d}\boldsymbol{l}_2\times(\mathrm{d}\boldsymbol{l}_1\times\hat{\boldsymbol{r}}_{12})]$ (7.120)	$\mathrm{d}\boldsymbol{l}_{1,2}$ $I_{1,2}$ $\mathrm{d}\boldsymbol{F}_2$ μ_0	line elements currents flowing along $\mathrm{d}\boldsymbol{l}_1$ and $\mathrm{d}\boldsymbol{l}_2$ force on $\mathrm{d}\boldsymbol{l}_2$ permeability of free space
Force on a current-carrying element in a magnetic field	$\mathrm{d}\boldsymbol{F}=I\,\mathrm{d}\boldsymbol{l}\times\boldsymbol{B}$ (7.121)	$\mathrm{d}\boldsymbol{l}$ $\boldsymbol{F}$ I $\boldsymbol{B}$	line element force current flowing along $\mathrm{d}\boldsymbol{l}$ magnetic flux density
Force on a charge (Lorentz force)	$\boldsymbol{F}=q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$ (7.122)	$\boldsymbol{E}$ $\boldsymbol{v}$	electric field charge velocity
Force on an electric dipole[a]	$\boldsymbol{F}=(\boldsymbol{p}\cdot\nabla)\boldsymbol{E}$ (7.123)	$\boldsymbol{p}$	electric dipole moment
Force on a magnetic dipole[b]	$\boldsymbol{F}=(\boldsymbol{m}\cdot\nabla)\boldsymbol{B}$ (7.124)	$\boldsymbol{m}$	magnetic dipole moment
Torque on an electric dipole	$\boldsymbol{G}=\boldsymbol{p}\times\boldsymbol{E}$ (7.125)	$\boldsymbol{G}$	torque
Torque on a magnetic dipole	$\boldsymbol{G}=\boldsymbol{m}\times\boldsymbol{B}$ (7.126)		
Torque on a current loop	$\boldsymbol{G}=I_{\mathrm{L}}\oint_{\mathrm{loop}}\boldsymbol{r}\times(\mathrm{d}\boldsymbol{l}_{\mathrm{L}}\times\boldsymbol{B})$ (7.127)	$\mathrm{d}\boldsymbol{l}_{\mathrm{L}}$ $\boldsymbol{r}$ I_{L}	line-element (of loop) position vector of $\mathrm{d}\boldsymbol{l}_{\mathrm{L}}$ current around loop





[a] $\boldsymbol{F}$ simplifies to $\nabla(\boldsymbol{p}\cdot\boldsymbol{E})$ if p is intrinsic, $\nabla(pE/2)$ if $\boldsymbol{p}$ is induced by $\boldsymbol{E}$ and the medium is isotropic.
[b] $\boldsymbol{F}$ simplifies to $\nabla(\boldsymbol{m}\cdot\boldsymbol{B})$ if m is intrinsic, $\nabla(mB/2)$ if $\boldsymbol{m}$ is induced by $\boldsymbol{B}$ and the medium is isotropic.

Electromagnetic energy

Electromagnetic field energy density (in free space)	$u=\frac{1}{2}\epsilon_0E^2+\frac{1}{2}\frac{B^2}{\mu_0}$ (7.128)	u energy density E electric field B magnetic flux density ϵ_0 permittivity of free space μ_0 permeability of free space
Energy density in media	$u=\frac{1}{2}(\boldsymbol{D}\cdot\boldsymbol{E}+\boldsymbol{B}\cdot\boldsymbol{H})$ (7.129)	$\boldsymbol{D}$ electric displacement $\boldsymbol{H}$ magnetic field strength c speed of light
Energy flow (Poynting) vector	$\boldsymbol{N}=\boldsymbol{E}\times\boldsymbol{H}$ (7.130)	$\boldsymbol{N}$ energy flow rate per unit area ⊥ to the flow direction
Mean flux density at a distance r from a short oscillating dipole	$\langle\boldsymbol{N}\rangle=\frac{\omega^4p_0^2\sin^2\theta}{32\pi^2\epsilon_0c^3r^3}\boldsymbol{r}$ (7.131)	p_0 amplitude of dipole moment $\boldsymbol{r}$ vector from dipole ($\gg$wavelength) θ angle between $\boldsymbol{p}$ and $\boldsymbol{r}$ ω oscillation frequency
Total mean power from oscillating dipole[a]	$W=\frac{\omega^4p_0^2/2}{6\pi\epsilon_0c^3}$ (7.132)	W total mean radiated power
Self-energy of a charge distribution	$U_{\text{tot}}=\frac{1}{2}\int_{\text{volume}}\phi(\boldsymbol{r})\rho(\boldsymbol{r})\,\mathrm{d}\tau$ (7.133)	U_{tot} total energy $\mathrm{d}\tau$ volume element $\boldsymbol{r}$ position vector of $\mathrm{d}\tau$ ϕ electrical potential ρ charge density
Energy of an assembly of capacitors[b]	$U_{\text{tot}}=\frac{1}{2}\sum_i\sum_j C_{ij}V_iV_j$ (7.134)	V_i potential of ith capacitor C_{ij} mutual capacitance between capacitors i and j
Energy of an assembly of inductors[c]	$U_{\text{tot}}=\frac{1}{2}\sum_i\sum_j L_{ij}I_iI_j$ (7.135)	L_{ij} mutual inductance between inductors i and j
Intrinsic dipole in an electric field	$U_{\text{dip}}=-\boldsymbol{p}\cdot\boldsymbol{E}$ (7.136)	U_{dip} energy of dipole $\boldsymbol{p}$ electric dipole moment
Intrinsic dipole in a magnetic field	$U_{\text{dip}}=-\boldsymbol{m}\cdot\boldsymbol{B}$ (7.137)	$\boldsymbol{m}$ magnetic dipole moment
Hamiltonian of a charged particle in an EM field[d]	$H=\frac{\lvert\boldsymbol{p}_{\boldsymbol{m}}-q\boldsymbol{A}\rvert^2}{2m}+q\phi$ (7.138)	H Hamiltonian $\boldsymbol{p}_{\boldsymbol{m}}$ particle momentum q particle charge m particle mass $\boldsymbol{A}$ magnetic vector potential

[a] Sometimes called "Larmor's formula."
[b] C_{ii} is the self-capacitance of the ith body. Note that $C_{ij}=C_{ji}$.
[c] L_{ii} is the self-inductance of the ith body. Note that $L_{ij}=L_{ji}$.
[d] Newtonian limit, i.e., velocity $\ll c$.