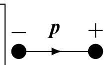
7.4 Fields associated with media

Polarisation

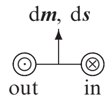


Definition of electric dipole moment	$\boldsymbol{p}=q\boldsymbol{a}$	(7.80)	$\pm q$ end charges $\boldsymbol{a}$ charge separation vector (from − to +)
Generalised electric dipole moment	$\boldsymbol{p}=\int_{\text{volume}}\boldsymbol{r}'\rho\,\mathrm{d}\tau'$	(7.81)	$\boldsymbol{p}$ dipole moment ρ charge density $\mathrm{d}\tau'$ volume element $\boldsymbol{r}'$ vector to $\mathrm{d}\tau'$
Electric dipole potential	$\phi(\boldsymbol{r})=\dfrac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\epsilon_0 r^3}$	(7.82)	ϕ dipole potential $\boldsymbol{r}$ vector from dipole ϵ_0 permittivity of free space
Dipole moment per unit volume (polarisation)[a]	$\boldsymbol{P}=n\boldsymbol{p}$	(7.83)	$\boldsymbol{P}$ polarisation n number of dipoles per unit volume
Induced volume charge density	$\nabla\cdot\boldsymbol{P}=-\rho_{\text{ind}}$	(7.84)	ρ_{ind} volume charge density
Induced surface charge density	$\sigma_{\text{ind}}=\boldsymbol{P}\cdot\hat{\boldsymbol{s}}$	(7.85)	σ_{ind} surface charge density $\hat{\boldsymbol{s}}$ unit normal to surface
Definition of electric displacement	$\boldsymbol{D}=\epsilon_0\boldsymbol{E}+\boldsymbol{P}$	(7.86)	$\boldsymbol{D}$ electric displacement $\boldsymbol{E}$ electric field
Definition of electric susceptibility	$\boldsymbol{P}=\epsilon_0\chi_E\boldsymbol{E}$	(7.87)	χ_E electrical susceptibility (may be a tensor)
Definition of relative permittivity[b]	$\epsilon_{\text{r}}=1+\chi_E$ $\boldsymbol{D}=\epsilon_0\epsilon_{\text{r}}\boldsymbol{E}$ $=\epsilon\boldsymbol{E}$	(7.88) (7.89) (7.90)	ϵ_{r} relative permittivity ϵ permittivity
Atomic polarisability[c]	$\boldsymbol{p}=\alpha\boldsymbol{E}_{\text{loc}}$	(7.91)	α polarisability $\boldsymbol{E}_{\text{loc}}$ local electric field
Depolarising fields	$\boldsymbol{E}_{\text{d}}=-\dfrac{N_{\text{d}}\boldsymbol{P}}{\epsilon_0}$	(7.92)	$\boldsymbol{E}_{\text{d}}$ depolarising field N_{d} depolarising factor =1/3 (sphere) =1 (thin slab $\perp$ to $\boldsymbol{P}$) =0 (thin slab $\parallel$ to $\boldsymbol{P}$) =1/2 (long circular cylinder, axis $\perp$ to $\boldsymbol{P}$)
Clausius–Mossotti equation[d]	$\dfrac{n\alpha}{3\epsilon_0}=\dfrac{\epsilon_{\text{r}}-1}{\epsilon_{\text{r}}+2}$	(7.93)	

[a]Assuming dipoles are parallel. The equivalent of Equation (7.112) holds for a hot gas of electric dipoles.
[b]Relative permittivity as defined here is for a linear isotropic medium.
[c]The polarisability of a conducting sphere radius a is $\alpha=4\pi\epsilon_0 a^3$. The definition $\boldsymbol{p}=\alpha\epsilon_0\boldsymbol{E}_{\text{loc}}$ is also used.
[d]With the substitution $\eta^2=\epsilon_{\text{r}}$ [cf. Equation (7.195) with $\mu_{\text{r}}=1$] this is also known as the “Lorentz–Lorenz formula.”

Magnetisation

Definition of magnetic dipole moment	$\mathrm{d}\boldsymbol{m} = I\,\mathrm{d}\boldsymbol{s}$	(7.94)	$\mathrm{d}\boldsymbol{m}$ dipole moment I loop current $\mathrm{d}\boldsymbol{s}$ loop area (right-hand sense with respect to loop current)
Generalised magnetic dipole moment	$\boldsymbol{m} = \frac{1}{2}\int_{\text{volume}} \boldsymbol{r}' \times \boldsymbol{J}\,\mathrm{d}\tau'$	(7.95)	$\boldsymbol{m}$ dipole moment $\boldsymbol{J}$ current density $\mathrm{d}\tau'$ volume element $\boldsymbol{r}'$ vector to $d\tau'$
Magnetic dipole (scalar) potential	$\phi_{\mathrm{m}}(\boldsymbol{r}) = \frac{\mu_0 \boldsymbol{m}\cdot\boldsymbol{r}}{4\pi r^3}$	(7.96)	ϕ_{m} magnetic scalar potential $\boldsymbol{r}$ vector from dipole μ_0 permeability of free space
Dipole moment per unit volume (magnetisation)[a]	$\boldsymbol{M} = n\boldsymbol{m}$	(7.97)	$\boldsymbol{M}$ magnetisation n number of dipoles per unit volume
Induced volume current density	$\boldsymbol{J}_{\mathrm{ind}} = \nabla \times \boldsymbol{M}$	(7.98)	$\boldsymbol{J}_{\mathrm{ind}}$ volume current density (i.e., A m^{-2})
Induced surface current density	$\boldsymbol{j}_{\mathrm{ind}} = \boldsymbol{M} \times \hat{\boldsymbol{s}}$	(7.99)	$\boldsymbol{j}_{\mathrm{ind}}$ surface current density (i.e., A m^{-1}) $\hat{\boldsymbol{s}}$ unit normal to surface
Definition of magnetic field strength, $\boldsymbol{H}$	$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$	(7.100)	$\boldsymbol{B}$ magnetic flux density $\boldsymbol{H}$ magnetic field strength
Definition of magnetic susceptibility	$\boldsymbol{M} = \chi_H \boldsymbol{H}$ $= \frac{\chi_B \boldsymbol{B}}{\mu_0}$ $\chi_B = \frac{\chi_H}{1+\chi_H}$	(7.101) (7.102) (7.103)	χ_H magnetic susceptibility. χ_B is also used (both may be tensors)
Definition of relative permeability[b]	$\boldsymbol{B} = \mu_0 \mu_{\mathrm{r}} \boldsymbol{H}$ $= \mu \boldsymbol{H}$ $\mu_{\mathrm{r}} = 1 + \chi_H$ $= \frac{1}{1-\chi_B}$	(7.104) (7.105) (7.106) (7.107)	μ_{r} relative permeability μ permeability



[a]Assuming all the dipoles are parallel. See Equation (7.112) for a classical paramagnetic gas and page 101 for the quantum generalisation.
[b]Relative permeability as defined here is for a linear isotropic medium.

Paramagnetism and diamagnetism

Diamagnetic moment of an atom	$\boldsymbol{m} = -\frac{e^2}{6m_e} Z \langle r^2 \rangle \boldsymbol{B}$	(7.108)
Intrinsic electron magnetic moment[a]	$\boldsymbol{m} \simeq -\frac{e}{2m_e} g \boldsymbol{J}$	(7.109)
Langevin function	$\mathscr{L}(x) = \coth x - \frac{1}{x}$	(7.110)
	$\simeq x/3 \qquad (x \lesssim 1)$	(7.111)
Classical gas paramagnetism ($\lvert\boldsymbol{J}\rvert \gg \hbar$)	$\langle M \rangle = n m_0 \mathscr{L}\left(\frac{m_0 B}{kT}\right)$	(7.112)
Curie's law	$\chi_H = \frac{\mu_0 n m_0^2}{3kT}$	(7.113)
Curie–Weiss law	$\chi_H = \frac{\mu_0 n m_0^2}{3k(T - T_c)}$	(7.114)

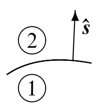
$\boldsymbol{m}$	magnetic moment
$\langle r^2 \rangle$	mean squared orbital radius (of all electrons)
Z	atomic number
$\boldsymbol{B}$	magnetic flux density
m_e	electron mass
$-e$	electronic charge
$\boldsymbol{J}$	total angular momentum
g	Landé g-factor (=2 for spin, =1 for orbital momentum)
$\mathscr{L}(x)$	Langevin function
$\langle M \rangle$	apparent magnetisation
m_0	magnitude of magnetic dipole moment
n	dipole number density
T	temperature
k	Boltzmann constant
χ_H	magnetic susceptibility
μ_0	permeability of free space
T_c	Curie temperature

[a] See also page 100.

Boundary conditions for $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{B}$, and $\boldsymbol{H}$[a]

Parallel component of the electric field	$E_\parallel$ continuous	(7.115)
Perpendicular component of the magnetic flux density	$B_\perp$ continuous	(7.116)
Electric displacement[b]	$\hat{\boldsymbol{s}} \cdot (\boldsymbol{D}_2 - \boldsymbol{D}_1) = \sigma$	(7.117)
Magnetic field strength[c]	$\hat{\boldsymbol{s}} \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{j}_s$	(7.118)

$\parallel$	component parallel to interface
$\perp$	component perpendicular to interface
$\boldsymbol{D}_{1,2}$	electrical displacements in media 1 & 2
$\hat{\boldsymbol{s}}$	unit normal to surface, directed $1 \rightarrow 2$
σ	surface density of free charge
$\boldsymbol{H}_{1,2}$	magnetic field strengths in media 1 & 2
$\boldsymbol{j}_s$	surface current per unit width



[a] At the plane surface between two uniform media.
[b] If $\sigma = 0$, then $D_\perp$ is continuous.
[c] If $\boldsymbol{j}_s = \boldsymbol{0}$ then $H_\parallel$ is continuous.