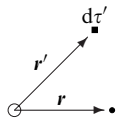


7.3 Electromagnetic fields (general)

Field relationships

Conservation of charge	$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$	(7.39)	$\mathbf{J}$ current density ρ charge density t time
Magnetic vector potential	$\mathbf{B} = \nabla \times \mathbf{A}$	(7.40)	$\mathbf{A}$ vector potential
Electric field from potentials	$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$	(7.41)	ϕ electrical potential
Coulomb gauge condition	$\nabla \cdot \mathbf{A} = 0$	(7.42)	
Lorenz gauge condition	$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$	(7.43)	c speed of light
Potential field equations ^a	$\frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = \frac{\rho}{\epsilon_0}$	(7.44)	
	$\frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}$	(7.45)	
Expression for ϕ in terms of ρ^a	$\phi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\rho(\mathbf{r}', t - \mathbf{r} - \mathbf{r}' /c)}{ \mathbf{r} - \mathbf{r}' } d\tau'$	(7.46)	$d\tau'$ volume element $\mathbf{r}'$ position vector of $d\tau'$
Expression for $\mathbf{A}$ in terms of $\mathbf{J}^a$	$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int_{\text{volume}} \frac{\mathbf{J}(\mathbf{r}', t - \mathbf{r} - \mathbf{r}' /c)}{ \mathbf{r} - \mathbf{r}' } d\tau'$	(7.47)	μ_0 permeability of free space

^aAssumes the Lorenz gauge.

Liénard–Wiechert potentials^a

Electrical potential of a moving point charge	$\phi = \frac{q}{4\pi\epsilon_0(\mathbf{r} - \mathbf{v} \cdot \mathbf{r}/c)}$	(7.48)	q charge $\mathbf{r}$ vector from charge to point of observation $\mathbf{v}$ particle velocity
Magnetic vector potential of a moving point charge	$\mathbf{A} = \frac{\mu_0 q \mathbf{v}}{4\pi(\mathbf{r} - \mathbf{v} \cdot \mathbf{r}/c)}$	(7.49)	

^aIn free space. The right-hand sides of these equations are evaluated at retarded times, i.e., at $t' = t - |\mathbf{r}'|/c$, where $\mathbf{r}'$ is the vector from the charge to the observation point at time t' .

Maxwell's equations

Differential form:	Integral form:
$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (7.50)$	$\oint_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_{\text{volume}} \rho \, d\tau \quad (7.51)$
$\nabla \cdot \mathbf{B} = 0 \quad (7.52)$	$\oint_{\text{closed surface}} \mathbf{B} \cdot d\mathbf{s} = 0 \quad (7.53)$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (7.54)$	$\oint_{\text{loop}} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \quad (7.55)$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (7.56)$	$\oint_{\text{loop}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \epsilon_0 \int_{\text{surface}} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{s} \quad (7.57)$
Equation (7.51) is “Gauss’s law” Equation (7.55) is “Faraday’s law” $\mathbf{E}$ electric field $\mathbf{B}$ magnetic flux density $\mathbf{J}$ current density ρ charge density	$d\mathbf{s}$ surface element $d\tau$ volume element $d\mathbf{l}$ line element Φ linked magnetic flux ($= \int \mathbf{B} \cdot d\mathbf{s}$) I linked current ($= \int \mathbf{J} \cdot d\mathbf{s}$) t time

Maxwell's equations (using $\mathbf{D}$ and $\mathbf{H}$)

Differential form:	Integral form:
$\nabla \cdot \mathbf{D} = \rho_{\text{free}} \quad (7.58)$	$\oint_{\text{closed surface}} \mathbf{D} \cdot d\mathbf{s} = \int_{\text{volume}} \rho_{\text{free}} \, d\tau \quad (7.59)$
$\nabla \cdot \mathbf{B} = 0 \quad (7.60)$	$\oint_{\text{closed surface}} \mathbf{B} \cdot d\mathbf{s} = 0 \quad (7.61)$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (7.62)$	$\oint_{\text{loop}} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \quad (7.63)$
$\nabla \times \mathbf{H} = \mathbf{J}_{\text{free}} + \frac{\partial \mathbf{D}}{\partial t} \quad (7.64)$	$\oint_{\text{loop}} \mathbf{H} \cdot d\mathbf{l} = I_{\text{free}} + \int_{\text{surface}} \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} \quad (7.65)$
$\mathbf{D}$ displacement field ρ_{free} free charge density (in the sense of $\rho = \rho_{\text{induced}} + \rho_{\text{free}}$) $\mathbf{B}$ magnetic flux density $\mathbf{H}$ magnetic field strength $\mathbf{J}_{\text{free}}$ free current density (in the sense of $\mathbf{J} = \mathbf{J}_{\text{induced}} + \mathbf{J}_{\text{free}}$)	$\mathbf{E}$ electric field $d\mathbf{s}$ surface element $d\tau$ volume element $d\mathbf{l}$ line element Φ linked magnetic flux ($= \int \mathbf{B} \cdot d\mathbf{s}$) I_{free} linked free current ($= \int \mathbf{J}_{\text{free}} \cdot d\mathbf{s}$) t time

Relativistic electrodynamics

Lorentz transformation of electric and magnetic fields	$E'_{\parallel} = E_{\parallel}$	(7.66)	E	electric field
	$E'_{\perp} = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp}$	(7.67)	$\mathbf{B}$	magnetic flux density
	$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$	(7.68)	$'$	measured in frame moving at relative velocity $\mathbf{v}$
	$\mathbf{B}'_{\perp} = \gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E}/c^2)_{\perp}$	(7.69)	γ	Lorentz factor $= [1 - (v/c)^2]^{-1/2}$
Lorentz transformation of current and charge densities	$\rho' = \gamma(\rho - vJ_{\parallel}/c^2)$	(7.70)	$\parallel$	parallel to $\mathbf{v}$
	$J'_{\perp} = J_{\perp}$	(7.71)	$\perp$	perpendicular to $\mathbf{v}$
	$J'_{\parallel} = \gamma(J_{\parallel} - v\rho)$	(7.72)	$\mathbf{J}$	current density
Lorentz transformation of potential fields	$\phi' = \gamma(\phi - vA_{\parallel})$	(7.73)	ρ	charge density
	$A'_{\perp} = A_{\perp}$	(7.74)	ϕ	electric potential
	$A'_{\parallel} = \gamma(A_{\parallel} - v\phi/c^2)$	(7.75)	$\mathbf{A}$	magnetic vector potential
Four-vector fields ^a	$\tilde{\mathbf{J}} = (\rho c, \mathbf{J})$	(7.76)		
	$\tilde{\mathbf{A}} = \left(\frac{\phi}{c}, \mathbf{A} \right)$	(7.77)	$\tilde{\mathbf{J}}$	current density four-vector
	$\square^2 = \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2}, -\nabla^2 \right)$	(7.78)	$\tilde{\mathbf{A}}$	potential four-vector
	$\square^2 \tilde{\mathbf{A}} = \mu_0 \tilde{\mathbf{J}}$	(7.79)	$\square^2$	D'Alembertian operator

^aOther sign conventions are common here. See page 65 for a general definition of four-vectors.