4.7 High energy and nuclear physics

Nuclear decay

Nuclear decay law	$$N(t) = N(0)e^{-\lambda t} \quad (4.163)$$	$N(t)$ number of nuclei remaining after time t t time
Half-life and mean life	$$T_{1/2} = \frac{\ln 2}{\lambda} \quad (4.164)$$ $$\langle T\rangle = 1/\lambda \quad (4.165)$$	λ decay constant $T_{1/2}$ half-life $\langle T\rangle$ mean lifetime
Successive decays $1 \rightarrow 2 \rightarrow 3$ (species 3 stable)	$$N_1(t) = N_1(0)e^{-\lambda_1 t} \quad (4.166)$$ $$N_2(t) = N_2(0)e^{-\lambda_2 t} + \frac{N_1(0)\lambda_1(e^{-\lambda_1 t} - e^{-\lambda_2 t})}{\lambda_2 - \lambda_1} \quad (4.167)$$ $$N_3(t) = N_3(0) + N_2(0)(1 - e^{-\lambda_2 t}) + N_1(0)\left(1 + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}\right) \quad (4.168)$$	N_1 population of species 1 N_2 population of species 2 N_3 population of species 3 λ_1 decay constant $1 \rightarrow 2$ λ_2 decay constant $2 \rightarrow 3$
Geiger's law[a]	$$v^3 = a(R - x) \quad (4.169)$$	v velocity of α particle x distance from source a constant
Geiger–Nuttall rule	$$\log\lambda = b + c\log R \quad (4.170)$$	R range b, c constants for each series α, β, and γ

[a] For α particles in air (empirical).

Nuclear binding energy

Liquid drop model[a]	$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta(A) \quad (4.171)$$ $$\delta(A) \simeq \begin{cases} +a_p A^{-3/4} & Z, N \text{ both even} \\ -a_p A^{-3/4} & Z, N \text{ both odd} \\ 0 & \text{otherwise} \end{cases} \quad (4.172)$$	N number of neutrons A mass number ($= N + Z$) B semi-empirical binding energy Z number of protons a_v volume term ($\sim$ 15.8 MeV) a_s surface term ($\sim$ 18.0 MeV) a_c Coulomb term ($\sim$ 0.72 MeV) a_a asymmetry term ($\sim$ 23.5 MeV) a_p pairing term ($\sim$ 33.5 MeV)
Semi-empirical mass formula	$$M(Z,A) = ZM_H + Nm_n - B \quad (4.173)$$	$M(Z,A)$ atomic mass M_H mass of hydrogen atom m_n neutron mass

[a] Coefficient values are empirical and approximate.

Nuclear collisions

Breit–Wigner formula[a]	$$\sigma(E)=\frac{\pi}{k^2}g\frac{\Gamma_{ab}\Gamma_c}{(E-E_0)^2+\Gamma^2/4} \quad (4.174)$$ $$g=\frac{2J+1}{(2s_a+1)(2s_b+1)} \quad (4.175)$$	$\sigma(E)$ cross-section for $a+b\rightarrow c$; k incoming wavenumber; g spin factor; E total energy (PE + KE); E_0 resonant energy; Γ width of resonant state R; Γ_{ab} partial width into $a+b$; Γ_c partial width into c
Total width	$$\Gamma=\Gamma_{ab}+\Gamma_c \quad (4.176)$$	
Resonance lifetime	$$\tau=\frac{\hbar}{\Gamma} \quad (4.177)$$	τ resonance lifetime; J total angular momentum quantum number of R; $s_{a,b}$ spins of a and b
Born scattering formula[b]	$$\frac{d\sigma}{d\Omega}=\left\|\frac{2\mu}{\hbar^2}\int_0^\infty\frac{\sin Kr}{Kr}V(r)r^2\,dr\right\|^2 \quad (4.178)$$	$\frac{d\sigma}{d\Omega}$ differential collision cross-section; μ reduced mass; K $=\|\boldsymbol{k}_{in}-\boldsymbol{k}_{out}\|$ (see footnote); r radial distance; $V(r)$ potential energy of interaction
Mott scattering formula[c]	$$\frac{d\sigma}{d\Omega}=\left(\frac{\alpha}{4E}\right)^2\left[\csc^4\frac{\chi}{2}+\sec^4\frac{\chi}{2}+\frac{A\cos\left(\frac{\alpha}{\hbar v}\ln\tan^2\frac{\chi}{2}\right)}{\sin^2\frac{\chi}{2}\cos\frac{\chi}{2}}\right] \quad (4.179)$$ $$\frac{d\sigma}{d\Omega}\simeq\left(\frac{\alpha}{2E}\right)^2\frac{4-3\sin^2\chi}{\sin^4\chi} \quad (A=-1,\alpha\ll v\hbar) \quad (4.180)$$	$\hbar$ (Planck constant)/2π; α/r scattering potential energy; χ scattering angle; v closing velocity; A $=2$ for spin-zero particles, $=-1$ for spin-half particles

[a]For the reaction $a+b\leftrightarrow R\rightarrow c$ in the centre of mass frame.
[b]For a central field. The Born approximation holds when the potential energy of scattering, V, is much less than the total kinetic energy. K is the magnitude of the change in the particle's wavevector due to scattering.
[c]For identical particles undergoing Coulomb scattering in the centre of mass frame. Nonidentical particles obey the Rutherford scattering formula (page 72).

Relativistic wave equations[a]

Klein–Gordon equation (massive, spin zero particles)	$$(\nabla^2-m^2)\psi=\frac{\partial^2\psi}{\partial t^2} \quad (4.181)$$	ψ wavefunction; m particle mass; t time
Weyl equations (massless, spin 1/2 particles)	$$\frac{\partial\boldsymbol{\psi}}{\partial t}=\pm\left(\boldsymbol{\sigma}_x\frac{\partial\boldsymbol{\psi}}{\partial x}+\boldsymbol{\sigma}_y\frac{\partial\boldsymbol{\psi}}{\partial y}+\boldsymbol{\sigma}_z\frac{\partial\boldsymbol{\psi}}{\partial z}\right) \quad (4.182)$$	$\boldsymbol{\psi}$ spinor wavefunction; $\boldsymbol{\sigma}_i$ Pauli spin matrices (see page 26)
Dirac equation (massive, spin 1/2 particles)	$$(\mathbf{i}\gamma^\mu\partial\mu-m)\boldsymbol{\psi}=0 \quad (4.183)$$ where $$\partial\mu=\left(\frac{\partial}{\partial t},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) \quad (4.184)$$ $$(\gamma^0)^2=\mathbf{1}_4;\quad(\gamma^1)^2=(\gamma^2)^2=(\gamma^3)^2=-\mathbf{1}_4 \quad (4.185)$$	$\mathbf{i}$ $\mathbf{i}^2=-1$; γ^μ Dirac matrices: $\gamma^0=\begin{pmatrix}\mathbf{1}_2 & 0\\ 0 & -\mathbf{1}_2\end{pmatrix}$, $\gamma^i=\begin{pmatrix}0 & \boldsymbol{\sigma}_i\\ -\boldsymbol{\sigma}_i & 0\end{pmatrix}$; $\mathbf{1}_n$ $n\times n$ unit matrix

[a]Written in natural units, with $c=\hbar=1$.