2.10 Roots of quadratic and cubic equations

Quadratic equations

Equation	$ax^2+bx+c=0 \qquad (a\neq 0)$	(2.454)	x variable a,b,c real constants
Solutions	$x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	(2.455)	x_1,x_2 quadratic roots
	$=\dfrac{-2c}{b\pm\sqrt{b^2-4ac}}$	(2.456)	
Solution combinations	$x_1+x_2=-b/a$	(2.457)	
	$x_1x_2=c/a$	(2.458)	

Cubic equations

Equation	$ax^3+bx^2+cx+d=0 \quad (a\neq 0)$	(2.459)	x variable a,b,c,d real constants
Intermediate definitions	$p=\dfrac{1}{3}\left(\dfrac{3c}{a}-\dfrac{b^2}{a^2}\right)$	(2.460)	
	$q=\dfrac{1}{27}\left(\dfrac{2b^3}{a^3}-\dfrac{9bc}{a^2}+\dfrac{27d}{a}\right)$	(2.461)	D discriminant
	$D=\left(\dfrac{p}{3}\right)^3+\left(\dfrac{q}{2}\right)^2$	(2.462)	

If $D\geq 0$, also define:

$$u=\left(\frac{-q}{2}+D^{1/2}\right)^{1/3} \qquad (2.463)$$

$$v=\left(\frac{-q}{2}-D^{1/2}\right)^{1/3} \qquad (2.464)$$

$$y_1=u+v \qquad (2.465)$$

$$y_{2,3}=\frac{-(u+v)}{2}\pm \mathbf{i}\frac{u-v}{2}3^{1/2} \qquad (2.466)$$

1 real, 2 complex roots
(if $D=0$: 3 real roots, at least 2 equal)

If $D<0$, also define:

$$\phi=\arccos\left[\frac{-q}{2}\left(\frac{|p|}{3}\right)^{-3/2}\right] \qquad (2.467)$$

$$y_1=2\left(\frac{|p|}{3}\right)^{1/2}\cos\frac{\phi}{3} \qquad (2.468)$$

$$y_{2,3}=-2\left(\frac{|p|}{3}\right)^{1/2}\cos\frac{\phi\pm\pi}{3} \qquad (2.469)$$

3 distinct real roots

Solutions[a]	$x_n=y_n-\dfrac{b}{3a}$	(2.470)	x_n cubic roots ($n=1,2,3$)
Solution combinations	$x_1+x_2+x_3=-b/a$	(2.471)	
	$x_1x_2+x_1x_3+x_2x_3=c/a$	(2.472)	
	$x_1x_2x_3=-d/a$	(2.473)	

[a] y_n are solutions to the reduced equation $y^3+py+q=0$.