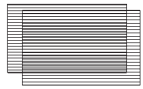
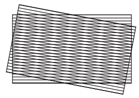
2.6 Mensuration

Moiré fringes[a]

Parallel pattern	$d_{\mathrm{M}}=\left\lvert\frac{1}{d_1}-\frac{1}{d_2}\right\rvert^{-1}$	(2.244)	d_{M} Moiré fringe spacing $d_{1,2}$ grating spacings d common grating spacing θ relative rotation angle ($\lvert\theta\rvert\le\pi/2$)
Rotational pattern[b]	$d_{\mathrm{M}}=\frac{d}{2\lvert\sin(\theta/2)\rvert}$	(2.245)	

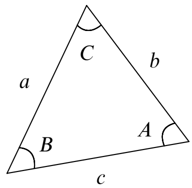




[a]From overlapping linear gratings.
[b]From identical gratings, spacing d, with a relative rotation θ.

Plane triangles

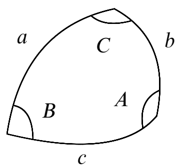
Sine formula[a]	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$	(2.246)
Cosine formulas	$a^2=b^2+c^2-2bc\cos A$	(2.247)
	$\cos A=\frac{b^2+c^2-a^2}{2bc}$	(2.248)
	$a=b\cos C+c\cos B$	(2.249)
Tangent formula	$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$	(2.250)
Area	area $=\frac{1}{2}ab\sin C$	(2.251)
	$=\frac{a^2}{2}\frac{\sin B\sin C}{\sin A}$	(2.252)
	$=[s(s-a)(s-b)(s-c)]^{1/2}$	(2.253)
	where $s=\frac{1}{2}(a+b+c)$	(2.254)



[a]The diameter of the circumscribed circle equals $a/\sin A$.

Spherical triangles[a]

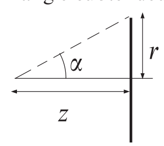
Sine formula	$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$	(2.255)
Cosine formulas	$\cos a=\cos b\cos c+\sin b\sin c\cos A$	(2.256)
	$\cos A=-\cos B\cos C+\sin B\sin C\cos a$	(2.257)
Analogue formula	$\sin a\cos B=\cos b\sin c-\sin b\cos c\cos A$	(2.258)
Four-parts formula	$\cos a\cos C=\sin a\cot b-\sin C\cot B$	(2.259)
Area[b]	$E=A+B+C-\pi$	(2.260)



[a]On a unit sphere.
[b]Also called the "spherical excess."

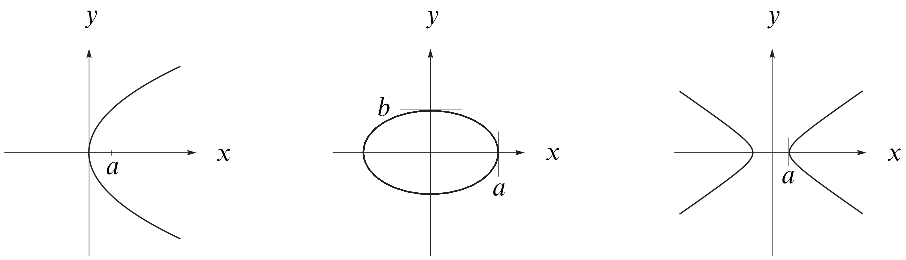
Perimeter, area, and volume

Perimeter of circle	$P=2\pi r$	(2.261)	P r	perimeter radius
Area of circle	$A=\pi r^2$	(2.262)	A	area
Surface area of sphere[a]	$A=4\pi R^2$	(2.263)	R	sphere radius
Volume of sphere	$V=\frac{4}{3}\pi R^3$	(2.264)	V	volume
Perimeter of ellipse[b]	$P=4a\,\mathrm{E}(\pi/2,e)$	(2.265)	a b E e	semi-major axis semi-minor axis elliptic integral of the second kind (p. 45) eccentricity ($=1-b^2/a^2$)
	$\simeq 2\pi\left(\frac{a^2+b^2}{2}\right)^{1/2}$	(2.266)		
Area of ellipse	$A=\pi ab$	(2.267)		
Volume of ellipsoid[c]	$V=4\pi\frac{abc}{3}$	(2.268)	c	third semi-axis
Surface area of cylinder	$A=2\pi r(h+r)$	(2.269)	h	height
Volume of cylinder	$V=\pi r^2 h$	(2.270)		
Area of circular cone[d]	$A=\pi rl$	(2.271)	l	slant height
Volume of cone or pyramid	$V=A_\mathrm{b}h/3$	(2.272)	A_b	base area
Surface area of torus	$A=\pi^2(r_1+r_2)(r_2-r_1)$	(2.273)	r_1 r_2	inner radius outer radius
Volume of torus	$V=\frac{\pi^2}{4}(r_2^2-r_1^2)(r_2-r_1)$	(2.274)		
Area[d] of spherical cap, depth d	$A=2\pi Rd$	(2.275)	d	cap depth
Volume of spherical cap, depth d	$V=\pi d^2\left(R-\frac{d}{3}\right)$	(2.276)	Ω z α	solid angle distance from centre half-angle subtended
Solid angle of a circle from a point on its axis, z from centre	$\Omega=2\pi\left[1-\frac{z}{(z^2+r^2)^{1/2}}\right]$	(2.277)		
	$=2\pi(1-\cos\alpha)$	(2.278)		



[a]Sphere defined by $x^2+y^2+z^2=R^2$.
[b]The approximation is exact when $e=0$ and $e\simeq 0.91$, giving a maximum error of 11% at $e=1$.
[c]Ellipsoid defined by $x^2/a^2+y^2/b^2+z^2/c^2=1$.
[d]Curved surface only.

Conic sections



	parabola	*ellipse*	*hyperbola*
equation	$y^2 = 4ax$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
parametric form	$x = t^2/(4a)$ $y = t$	$x = a\cos t$ $y = b\sin t$	$x = \pm a\cosh t$ $y = b\sinh t$
foci	$(a,0)$	$(\pm\sqrt{a^2-b^2},0)$	$(\pm\sqrt{a^2+b^2},0)$
eccentricity	$e = 1$	$e = \frac{\sqrt{a^2-b^2}}{a}$	$e = \frac{\sqrt{a^2+b^2}}{a}$
directrices	$x = -a$	$x = \pm\frac{a}{e}$	$x = \pm\frac{a}{e}$

Platonic solids[a]

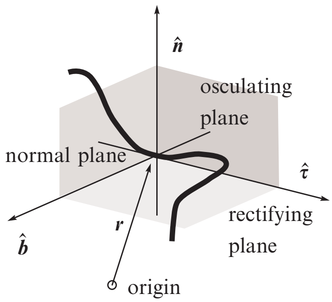
solid *(faces,edges,vertices)*	*volume*	*surface area*	*circumradius*	*inradius*
tetrahedron (4,6,4)	$\frac{a^3\sqrt{2}}{12}$	$a^2\sqrt{3}$	$\frac{a\sqrt{6}}{4}$	$\frac{a\sqrt{6}}{12}$
cube (6,12,8)	a^3	$6a^2$	$\frac{a\sqrt{3}}{2}$	$\frac{a}{2}$
octahedron (8,12,6)	$\frac{a^3\sqrt{2}}{3}$	$2a^2\sqrt{3}$	$\frac{a}{\sqrt{2}}$	$\frac{a}{\sqrt{6}}$
dodecahedron (12,30,20)	$\frac{a^3(15+7\sqrt{5})}{4}$	$3a^2\sqrt{5(5+2\sqrt{5})}$	$\frac{a}{4}\sqrt{3}(1+\sqrt{5})$	$\frac{a}{4}\sqrt{\frac{50+22\sqrt{5}}{5}}$
icosahedron (20,30,12)	$\frac{5a^3(3+\sqrt{5})}{12}$	$5a^2\sqrt{3}$	$\frac{a}{4}\sqrt{2(5+\sqrt{5})}$	$\frac{a}{4}\left(\sqrt{3}+\sqrt{\frac{5}{3}}\right)$

[a]Of side a. Both regular and irregular polyhedra follow the Euler relation, faces $-$ edges $+$ vertices $= 2$.

Curve measure

Length of plane curve	$$l=\int_a^b \left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{1/2}\mathrm{d}x$$	(2.279)	a b $y(x)$ l	start point end point plane curve length
Surface of revolution	$$A=2\pi\int_a^b y\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{1/2}\mathrm{d}x$$	(2.280)	A	surface area
Volume of revolution	$$V=\pi\int_a^b y^2\,\mathrm{d}x$$	(2.281)	V	volume
Radius of curvature	$$\rho=\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{3/2}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^{-1}$$	(2.282)	ρ	radius of curvature

Differential geometry[a]

Unit tangent	$$\hat{\boldsymbol{\tau}}=\frac{\dot{\boldsymbol{r}}}{\lvert\dot{\boldsymbol{r}}\rvert}=\frac{\dot{\boldsymbol{r}}}{v}$$	(2.283)	τ $\boldsymbol{r}$ v	tangent curve parameterised by $\boldsymbol{r}(t)$ $\lvert\dot{\boldsymbol{r}}(t)\rvert$
Unit principal normal	$$\hat{\boldsymbol{n}}=\frac{\ddot{\boldsymbol{r}}-\dot{v}\hat{\boldsymbol{\tau}}}{\lvert\ddot{\boldsymbol{r}}-\dot{v}\hat{\boldsymbol{\tau}}\rvert}$$	(2.284)	$\boldsymbol{n}$	principal normal
Unit binormal	$$\hat{\boldsymbol{b}}=\hat{\boldsymbol{\tau}}\times\hat{\boldsymbol{n}}$$	(2.285)	$\boldsymbol{b}$	binormal
Curvature	$$\kappa=\frac{\lvert\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}\rvert}{\lvert\dot{\boldsymbol{r}}\rvert^3}$$	(2.286)	κ	curvature
Radius of curvature	$$\rho=\frac{1}{\kappa}$$	(2.287)	ρ	radius of curvature
Torsion	$$\lambda=\frac{\dot{\boldsymbol{r}}\cdot(\ddot{\boldsymbol{r}}\times\dddot{\boldsymbol{r}})}{\lvert\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}\rvert^2}$$	(2.288)	λ	torsion
Frenet's formulas	$$\dot{\hat{\boldsymbol{\tau}}}=\kappa v\hat{\boldsymbol{n}}$$ (2.289) $$\dot{\hat{\boldsymbol{n}}}=-\kappa v\hat{\boldsymbol{\tau}}+\lambda v\hat{\boldsymbol{b}}$$ (2.290) $$\dot{\hat{\boldsymbol{b}}}=-\lambda v\hat{\boldsymbol{n}}$$ (2.291)			

[a]For a continuous curve in three dimensions, traced by the position vector $\boldsymbol{r}(t)$.