

ROTATORY MOTION

1. When a rigid body rotates about a fixed axis every particle of the body moves in circular path.
2. The perpendicular distance from axis of rotation to given point is called radius vector.
3. In rotation of a body about a fixed axis angular variables are same for all particles but linear variables changes
Relation between angular and linear variables.

1. $v = r\omega$

2. $a = r\alpha$

4. Kinematical Equations of rotatory Motion:

1. $\omega = \omega_0 + \alpha t$

2. $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

3. $\omega^2 - \omega_0^2 = 2\alpha\theta$

When ω_0 : Initial angular velocity ω : Final angular velocity α : Uniform angular acceleration

t : time

Important Points:

- 5.
- Torque:**

The turning effect of a force about the axis of rotation is called moment of force or torque.

Torque = Force x Perpendicular distance of line of action of force from axis of rotation.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = rF \sin \theta$$

 $\vec{r}$: Position vector

- 6.
- Couple:**

Two forces equal in magnitude but opposite in direction acting at two different points of a body constitute couple

- 7.
- Moment of Couple:**

The product of magnitude of force in couple and perpendicular distance between them.

- 8.
- Moment of Inertia:**

Def : Inability of a body to change its state of rotation by itself is called moment of inertia (I). Moment of inertia is analogue to mass in translatory motion.

Mathematical Definition:

Moment of inertia of a rigid body about a given axis of rotation is the sum of products of the masses of various particles and square of their perpendicular distance from the axis of rotation

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 \text{ or } I =$$

$$\sum_{i=1}^n m_i \cdot r_i^2$$

Units Kg.m^2 Dimensions: $\text{M L}^2 \text{T}^0$

9. Moment of inertia of different bodies

a. Rod – axis Passing through its centre

and perpendicular to its length $\frac{M \ell^2}{12}$

b. Rectangular plate or bar – axis Passing through its center and perpendicular to

plane $\frac{M \ell^2}{12} + \frac{M b^2}{12}$

c. Circular disc – axis Passign through its centre and perpendicular to its plane or

transverse axis $\frac{MR^2}{2}$

d. Solid cylinder – Its own axis $\frac{MR^2}{2}$

- 10.
- Radius of gyration:**

It is defined as the distance from the axis of rotation to a point where whole mass of the rotating body supposed to be concentrated.

It is denoted by 'K'

If 'K' is radius of gyration

Then $I = mK^2$ **Important Points:**

- a. Moment of inertia of a body depends on mass of the body and its distribution about the axis of rotation.
- b. Moment of inertia changes with change in position of axis of rotation
- c. Radius of gyration is not a constant quantity. Its value changes with change in location of axis of rotation.

Relation between τ and I . $\tau = I\alpha$.(It is analogue to $F = ma$ in translatory motion).

11. **Angular momentum (L):**

Moment of linear momentum of a particle in rotation about axis of rotation is known as Angular momentum

Angular momentum = linear momentum $\times$ Perpendicular distance from axis of rotation.
Angular momentum is a vector quantity

In vector form $\vec{L} = \vec{r} \times \vec{P}$

$$L = rp \sin \theta$$

$\vec{r}$: Position vector.

$r \sin \theta$: Perpendicular distance.

- Relation between L & I, $L = I\omega$.
- Linear momentum is measure of motion in linear motion.
- Angular momentum is the measure of motion in rotation. Newton's second law

for rotation $\vec{\tau} = \frac{d\vec{L}}{dt}$. Which gives rate

of change of angular momentum is directly proportional to torque.

- Relation between angular momentum and rotational K.E.

$$\therefore KE = \frac{L^2}{2I}$$

12. For a rolling body it possess both rotational and translational K.E.

$$E_K = E_t + E_r$$

$$E = \frac{1}{2}mr^2 + \frac{1}{2}I\omega^2$$

Where V: Velocity of centre of mass.

13. **Law of conservation of angular momentum:**

As long as external torque acting on a system is zero then the total angular momentum remains constant.

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\text{If } \vec{\tau} = 0$$

Where $\frac{d\vec{L}}{dt} = 0 \Rightarrow L$ is constant.

Where $L = I\omega$, $\therefore I\omega$ is constant

$$I_1 \omega_1 = I_2 \omega_2$$

14. **Theorems on moment of inertia****a. Perpendicular axes theorem:**

Statement: moment of inertia of a plane lamina about an axis perpendicular to its plane passing through a point is equal to the sum of moments of inertia of the lamina about any two mutually perpendicular axes in its plane and passing through same point.

$$I_z = I_x + I_y$$

b. Parallel axes theorem:

Moment of inertia of a rigid body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and square of the perpendicular distance between the two axes.

$$I_z = I_{cm} + MR^2$$

15. **Motion of a body in vertical circular plane:****Important Formula:**

- Minimum velocity at lowest point to describe vertical circular motion

$$V_1 = \sqrt{5Rg}$$

- Critical velocity at highest point

$$V_2 = \sqrt{Rg}$$

- Tension in the rope at lowest point

$$T_1 = \frac{mv_1^2}{R} + mg$$

- Tension in the rope at the highest point

$$T_2 = \frac{mv_2^2}{R} - mg$$

- At critical velocity tension at highest point is zero