Matrices
Part - 1

Assertion-Reasoning MCQs

Directions (Q. Nos. 65-79) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

65. **Assertion (A)** A 2×2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = i \times j$, is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Reason (R) If A is a 4×2 matrix, then the elements in A is 5.

66. **Assertion (A)** The matrix

$A = \begin{bmatrix} 3 & -1 & 0 \\ 3/2 & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is rectangular matrix of order 3.

Reason (R) If $A = [a_{ij}]_{m \times 1}$, then A is column matrix.

67. **Assertion (A)** Scalar matrix

$A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$, where k is a scalar, is an identity matrix when $k = 1$.

Reason (R) Every identity matrix is not a scalar matrix.

68. **Assertion (A)** If $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$, then $(-A)$ is given by $\begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$.

Reason (R) The negative of a matrix is given by $-A$ and is defined as $-A = (-1)A$.

69. Assertion (A) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then $3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$.

Reason (R) If the matrices A and B are of same order, say $m \times n$, satisfy the commutative law, then $A + B = B + A$.

70. Assertion (A) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then $A + B = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$.

Reason (R) Two different matrices can be added only if they are of same order.

71. Assertion (A) If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then $x = 2$, $y = 2$, $z = -5$ and $w = 4$.

Reason (R) Two matrices are equal, if their orders are same and their corresponding elements are equal.

72. Assertion (A) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then AB and BA both are defined.

Reason (R) For the two matrices A and B, the product AB is defined, if number of columns in A is equal to the number of rows in B.

73. Let A, B and C are three matrices of same order.

Now, consider the following statements

Assertion (A)
If $A = B$, then $AC = BC$.

Reason (R)
If $AC = BC$, then $A = B$.

74. Assertion (A)
$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Reason (R)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

75. Assertion (A) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k such that $A^2 = kA - 2I$, is -1.

Reason (R) If A and B are square matrices of same order, then $(A + B)(A + B)$ is equal to $A^2 + AB + BA + B^2$.

76. For the matrices $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, consider the following statements

Assertion (A) $(A + B)' = A' - B'$

Reason (R) $(A - B)' = A' - B'$

77. Let A and B be two symmetric matrices of order 3.

Assertion (A) $A(BA)$ and $(AB)A$ are symmetric matrices.

Reason (R) AB is symmetric matrix, if matrix multiplication of A with B is commutative.

78. **Assertion (A)** If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$, then B is the inverse of A.

Reason (R) If A is a square matrix of order m and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is called the inverse of A.

79. **Assertion (A)** If $A = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then A^{-1} does not exist.

Reason (R) On using elementary column operations $C_2 \rightarrow C_2 - 2C_1$ in the following matrix equation

$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have

$$\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}.$$

ANSWER KEY

Assertion-Reasoning MCQs

65. (c) *66. (d)* *67. (c)* *68. (a)* *69. (b)* *70. (a)* *71. (a)* *72. (a)* *73. (c)* *74. (d)*
75. (d) *76. (d)* *77. (b)* *78. (a)* *79. (b)*

SOLUTION

65. **Assertion** In general, the matrix A of order 2×2 is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Now, $a_{ij} = i \times j$, $i = 1, 2$ and $j = 1, 2$

$\therefore \quad a_{11} = 1, a_{12} = 2, a_{21} = 2$ and $a_{22} = 4$

Thus, matrix A is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Reason If A is a 4×2 matrix, then A has $4 \times 2 = 8$ elements.

66. **Assertion** $A = \begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

Reason In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix.

67. A scalar matrix $A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ is an identity matrix when $k = 1$.

But every identity matrix is clearly a scalar matrix.

68. We define $-A = (-1)A$.

If $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$,

then $-A$ is given by

$$-A = (-1)A = (-1)\begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$$

69. $3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6-(-2) & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

70. The given matrices are $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$.

Then, $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

71. We have, $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$

On comparing both the matrices, we get

$$z + 5 = 0 \Rightarrow z = -5$$

$$4 = w \Rightarrow w = 4$$

$$x + y = 4 \text{ and } xy = 4 \Rightarrow y = \frac{4}{x}$$

$\therefore \quad x + \frac{4}{x} = 4 \Rightarrow x^2 + 4 = 4x$

$\Rightarrow \quad x^2 - 4x + 4 = 0$

$\Rightarrow \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$

$\Rightarrow \quad x = \frac{4 \pm \sqrt{16-16}}{2} = \frac{4}{2} = 2$

$\therefore \quad y = 4 - x = 4 - 2 = 2$

72. The given matrices are

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Order of $A = 2 \times 3$; Order of $B = 3 \times 2$

Since, number of columns in A is equal to the number of rows in B.

$\Rightarrow AB$ is defined.

Also, number of columns in B is equal to the number of rows in A.

$\therefore$ The product BA is also defined.

74. **Assertion** $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}=\begin{bmatrix}10-3 & 5-4\\12+21 & 6+28\end{bmatrix}$

$$=\begin{bmatrix}7 & 1\\33 & 34\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}=\begin{bmatrix}10+6 & -2+7\\15+24 & -3+28\end{bmatrix}$$

$$=\begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$$

Hence, $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$

Reason Here, $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$

$$=\begin{bmatrix}-1+0+6 & 1-2+9 & 0+2+12\\0+0+0 & 0+(-1)+0 & 0+1+0\\-1+0+0 & 1-1+0 & 0+1+0\end{bmatrix}$$

$$=\begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$$

and $\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

$$=\begin{bmatrix}-1+0+0 & -2+1+0 & -3+0+0\\0+0+1 & 0-1+1 & 0+0+0\\2+0+4 & 4+3+4 & 6+0+0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1 & -3\\1 & 0 & 0\\6 & 11 & 6\end{bmatrix}$$

Hence, $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$

$$\neq\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$$

75. **Assertion** Given, $A^2 = kA - 2I$

$\Rightarrow \quad AA = kA - 2I$

$\Rightarrow \quad \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}=k\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}9-8 & -6+4\\12-8 & -8+4\end{bmatrix}=\begin{bmatrix}3k & -2k\\4k & -2k\end{bmatrix}-\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}=\begin{bmatrix}3k-2 & -2k\\4k & -2k-2\end{bmatrix}$

By definition of equality of matrix, the given matrices are equal and their corresponding elements are equal.

Now, comparing the corresponding elements, we get

$3k - 2 = 1 \Rightarrow k = 1$

$\Rightarrow \quad -2k = -2 \Rightarrow k = 1$

$\Rightarrow \quad 4k = 4 \Rightarrow k = 1$

$\Rightarrow \quad -4 = -2k - 2 \Rightarrow k = 1$

Hence, $k = 1$.

Reason We have,

$(A+B)(A+B) = A(A+B) + B(A+B)$

$= A^2 + AB + BA + B^2$

76. $A=\begin{bmatrix}3 & -1 & 0\\4 & 2 & 1\end{bmatrix}$

$\therefore A+B=\begin{bmatrix}3 & -1 & 0\\4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\1 & 2 & 3\end{bmatrix}$

$$=\begin{bmatrix}2 & 1 & 1\\5 & 4 & 4\end{bmatrix}$$

Assertion $(A+B)'=\begin{bmatrix}2 & 5\\1 & 4\\1 & 4\end{bmatrix}$

Now, $A'-B'=\begin{bmatrix}3 & 4\\-1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\2 & 2\\1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$

$\therefore \quad (A+B)' \neq A' - B'$

Reason

Now, $A-B=\begin{bmatrix}3 & -1 & 0\\4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\1 & 2 & 3\end{bmatrix}$

$$=\begin{bmatrix}4 & -3 & -1\\3 & 0 & -2\end{bmatrix}$$

$$(A-B)'=\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$$

and $A'-B'=\begin{bmatrix}3 & 4\\-1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\2 & 2\\1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$

$\therefore \quad (A-B)' = A' - B'$

77. **Assertion** Since, A and B are symmetric matrices.

$\therefore\ A^T = A$ and $B^T = B$.

Now, to check $A(BA)$ is symmetric.

Consider $[A(BA)]^T = (BA)^T \cdot A^T = (A^T B^T) A^T$

$= (AB)A = A(BA)$

So, $[A(BA)]^T = A(BA)$

$\Rightarrow A(BA)$ is symmetric.

Similarly, $(AB)A$ is symmetric.

So, Assertion is true.

Reason Now, $(AB)' = B'A'$

$= BA$

This will be symmetric, if A and B is commutative i.e. $AB = BA$.

Hence, both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

78. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.

Then, $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Also, $BA = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6-6 \\ -2+2 & -3+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, B is the inverse of A.

79. **Assertion** We have, $A = IA$

i.e. $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1/5 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 0 & 1 \end{bmatrix} A$

$\left[\text{applying } R_1 \to \frac{1}{10} R_1\right]$

$\Rightarrow \begin{bmatrix} 1 & -1/5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 1/2 & 1 \end{bmatrix}$

[applying $R_2 \to R_2 + 5R_1$]

We have all zeroes in the second row of the left hand side matrix of above equation.

Therefore, A^{-1} does not exist.

Reason The given matrix equation is

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$\because$ The column transformation $C_2 \to C_2 - 2C_1$ is applied.

$\therefore$ This transformation is applied on LHS and on second matrix of RHS.

Thus, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.