

(Chapter 8)(Electromagnetic Waves)

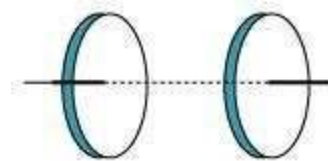
XII

Exercises

Question 8.1:

Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.

- (a) Calculate the capacitance and the rate of change of potential difference between the plates.
- (b) Obtain the displacement current across the plates.
- (c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



Answer

Radius of each circular plate, $r = 12 \text{ cm} = 0.12 \text{ m}$

Distance between the plates, $d = 5 \text{ cm} = 0.05 \text{ m}$

Charging current, $I = 0.15 \text{ A}$

Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

(a) Capacitance between the two plates is given by the relation,

$$C = \frac{\epsilon_0 A}{d}$$

Where,

$A = \text{Area of each plate} = \pi r^2$

$$C = \frac{\epsilon_0 \pi r^2}{d}$$

$$= \frac{8.85 \times 10^{-12} \times \pi \times (0.12)^2}{0.05}$$

$$= 8.0032 \times 10^{-12} \text{ F} = 80.032 \text{ pF}$$

Charge on each plate, $q = CV$

Where,

V = Potential difference across the plates

Differentiation on both sides with respect to time (t) gives:

$$\frac{dq}{dt} = C \frac{dV}{dt}$$

$$\text{But, } \frac{dq}{dt} = \text{current } (I)$$

$$\therefore \frac{dV}{dt} = \frac{I}{C}$$

$$\Rightarrow \frac{0.15}{80.032 \times 10^{-12}} = 1.87 \times 10^9 \text{ V/s}$$

Therefore, the change in potential difference between the plates is $1.87 \times 10^9 \text{ V/s}$.

(b) The displacement current across the plates is the same as the conduction current.

Hence, the displacement current, i_d is 0.15 A.

(c) Yes

Kirchhoff's first rule is valid at each plate of the capacitor provided that we take the sum of conduction and displacement for current.

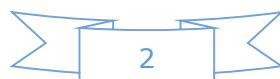
Question 8.2:

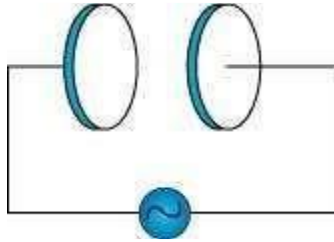
A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius $R = 6.0 \text{ cm}$ has a capacitance $C = 100 \text{ pF}$. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1} .

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.





Answer

Radius of each circular plate, $R = 6.0 \text{ cm} = 0.06 \text{ m}$

Capacitance of a parallel plate capacitor, $C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$

Supply voltage, $V = 230 \text{ V}$

Angular frequency, $\omega = 300 \text{ rad s}^{-1}$

(a) Rms value of conduction current, $I = \frac{V}{X_c}$

Where,

X_c = Capacitive reactance

$$= \frac{1}{\omega C}$$

$$\therefore I = V \times \omega C$$

$$= 230 \times 300 \times 100 \times 10^{-12}$$

$$= 6.9 \times 10^{-6} \text{ A}$$

$$= 6.9 \text{ } \mu\text{A}$$

Hence, the rms value of conduction current is $6.9 \text{ } \mu\text{A}$.

(b) Yes, conduction current is equal to displacement current.

(c) Magnetic field is given as:

$$B = \frac{\mu_0 r}{2\pi R^2} I_0 \quad \text{Where,}$$

$$\mu_0 = \text{Free space permeability} = 4\pi \times 10^{-7} \text{ N A}^{-2}$$

I_0 = Maximum value of current = $\sqrt{2} I$

r = Distance between the plates from the axis = 3.0 cm = 0.03 m .B

$$= \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi (0.06)^2}$$

$$= 1.63 \times 10^{-11} \text{ T}$$

Hence, the magnetic field at that point is $1.63 \times 10^{-11} \text{ T}$.

Question 8.3:

What physical quantity is the same for X-rays of wavelength 10^{-10} m , red light of wavelength $6800 \text{ \AA}$ and radiowaves of wavelength 500 m ?

Answer

The speed of light ($3 \times 10^8 \text{ m/s}$) in a vacuum is the same for all wavelengths. It is independent of the wavelength in the vacuum.

Question 8.4:

A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Answer

The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (H) are in the x-y plane. They are mutually perpendicular. Frequency of the wave, $\nu = 30 \text{ MHz} = 30 \times 10^6 \text{ s}^{-1}$

Speed of light in a vacuum, $c = 3 \times 10^8 \text{ m/s}$ Wavelength of a wave is given as:

$$\lambda = \frac{c}{\nu}$$
$$= \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}$$

Question 8.5:

A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Answer

A radio can tune to minimum frequency, $\nu_1 = 7.5 \text{ MHz} = 7.5 \times 10^6 \text{ Hz}$

Maximum frequency, $\nu_2 = 12 \text{ MHz} = 12 \times 10^6 \text{ Hz}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Corresponding wavelength for ν_1 can be calculated as:

$$\begin{aligned}\lambda_1 &= \frac{c}{\nu_1} \\ &= \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m}\end{aligned}$$

Corresponding wavelength for ν_2 can be calculated as:

$$\begin{aligned}\lambda_2 &= \frac{c}{\nu_2} \\ &= \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}\end{aligned}$$

Thus, the wavelength band of the radio is 40 m to 25 m.

Question 8.6:

A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz .

What is the frequency of the electromagnetic waves produced by the oscillator?

Answer

The frequency of an electromagnetic wave produced by the oscillator is the same as that of a charged particle oscillating about its mean position i.e., 10^9 Hz .

Question 8.7:

The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510 \text{ nT}$. What is the amplitude of the electric field part of the wave?

Answer

Amplitude of magnetic field of an electromagnetic wave in a vacuum,

$$B_0 = 510 \text{ nT} = 510 \times 10^{-9} \text{ T}$$

Speed of light in a vacuum, $c = 3 \times 10^8 \text{ m/s}$

Amplitude of electric field of the electromagnetic wave is given by the relation,

$$E = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153 \text{ N/C}$$

Therefore, the electric field part of the wave is 153 N/C .

Question 8.8:

Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120 \text{ N/C}$ and that its frequency is $\nu = 50.0 \text{ MHz}$. (a) Determine, B_0 , ω , k , and λ . (b) Find expressions for E and B .

Answer

Electric field amplitude, $E_0 = 120 \text{ N/C}$

Frequency of source, $\nu = 50.0 \text{ MHz} = 50 \times 10^6 \text{ Hz}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

(a) Magnitude of magnetic field strength is given as:

$$\begin{aligned} B_0 &= \frac{E_0}{c} \\ &= \frac{120}{3 \times 10^8} \\ &= 4 \times 10^{-7} \text{ T} = 400 \text{ nT} \end{aligned}$$

Angular frequency of source is given as:

$$\begin{aligned} \omega &= 2\pi\nu = 2\pi \times \\ &50 \times 10^6 \end{aligned}$$

$$= 3.14 \times 10^8 \text{ rad/s}$$

Propagation constant is given as:

$$k = \frac{\omega}{c}$$

$$= \frac{3.14 \times 10^8}{3 \times 10^8} = 1.05 \text{ rad/m}$$

Wavelength of wave is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{50 \times 10^6} = 6.0 \text{ m}$$

(b) Suppose the wave is propagating in the positive x direction. Then, the electric field vector will be in the positive y direction and the magnetic field vector will be in the positive z direction. This is because all three vectors are mutually perpendicular.

Equation of electric field vector is given as:

$$\vec{E} = E_0 \sin(kx - \omega t) \hat{j}$$

$$= 120 \sin[1.05x - 3.14 \times 10^8 t] \hat{j}$$

And, magnetic field vector is given as:

$$\vec{B} = B_0 \sin(kx - \omega t) \hat{k}$$

$$\vec{B} = (4 \times 10^{-7}) \sin[1.05x - 3.14 \times 10^8 t] \hat{k}$$

Question 8.9:

The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Answer

Energy of a photon is given as:

$$E = h\nu = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s λ =

Wavelength of radiation

$$\begin{aligned} \therefore E &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda} = \frac{19.8 \times 10^{-26}}{\lambda} \text{ J} \\ &= \frac{19.8 \times 10^{-26}}{\lambda \times 1.6 \times 10^{-19}} = \frac{12.375 \times 10^{-7}}{\lambda} \text{ eV} \end{aligned}$$

The given table lists the photon energies for different parts of an electromagnetic spectrum for different λ .

λ (m)	10^3	1	10^{-3}	10^{-6}	10^{-8}	10^{-10}	10^{-12}
E (eV)	12.375×10^{-10}	12.375×10^{-7}	12.375×10^{-4}	12.375×10^{-1}	12.375×10^1	12.375×10^3	12.375×10^5

The photon energies for the different parts of the spectrum of a source indicate the spacing of the relevant energy levels of the source.

Question 8.10:

In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1} .

- What is the wavelength of the wave?
- What is the amplitude of the oscillating magnetic field?
- Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^8 \text{ m s}^{-1}$.]

Answer

Frequency of the electromagnetic wave, $\nu = 2.0 \times 10^{10}$ Hz

Electric field amplitude, $E_0 = 48 \text{ V m}^{-1}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

(a) Wavelength of a wave is given as:

$$\begin{aligned}\lambda &= \frac{c}{\nu} \\ &= \frac{3 \times 10^8}{2 \times 10^{10}} = 0.015 \text{ m}\end{aligned}$$

(b) Magnetic field strength is given as:

$$\begin{aligned}B_0 &= \frac{E_0}{c} \\ &= \frac{48}{3 \times 10^8} = 1.6 \times 10^{-7} \text{ T}\end{aligned}$$

(c) Energy density of the electric field is given as:

$$U_E = \frac{1}{2} \epsilon_0 E^2$$

And, energy density of the magnetic field is given as:

$$U_B = \frac{1}{2\mu_0} B^2$$

Where,

ϵ_0 = Permittivity of free space

μ_0 = Permeability of free space

We have the relation connecting E and B as:

$$E = cB \dots (1)$$

Where,

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \dots (2)$$

Putting equation (2) in equation (1), we get

$$E = \frac{1}{\sqrt{\epsilon_0 \mu_0}} B$$

Squaring both sides, we get

$$E^2 = \frac{1}{\epsilon_0 \mu_0} B^2$$

$$\epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$\frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$\Rightarrow U_E = U_B$$