Polynomial

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An expression of the form p(x) = $a_0 + a_1x + a_2x^2 + \cdots \ldots\ldots\ldots\ldots + a_nx^n$ where $a_n \neq 0$ is called a polynomial in one variable x of degree n, where; $a_0, a_1, a_2 \ldots\ldots\ldots\ldots\ldots a_n$ are constants and they are called the coefficients of $x_0, x, x^2 \ldots\ldots\ldots x^n$. Each power of x is a non-negative integer.

Eg: -$2x^2 - 5x + 1$ is a polynomial of degree 2

Note: $\sqrt{x} + 3$ is not a polynomial

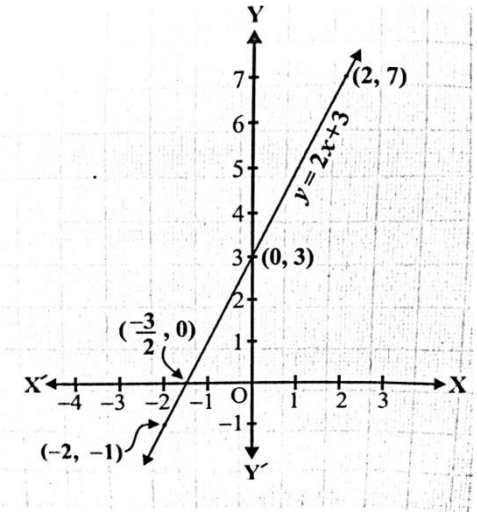
- A polynomial p(x) = $ax + b$ of degree 1 is called a linear polynomial Eg: $5x - 3, 2x$ etc
- A polynomial p(x) = $ax^2 + bx + c$ of degree 2 is called a quadratic polynomial Eg: $2x^2 + x - 1$
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial. Eg: $\sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1$ etc

Zeroes of a polynomial: A real number k is called a zero of polynomial p(x) if p(k)=0. If the graph of y= p(x) intersects the X-axis at n times, the number of zeroes of y= p(x) is n.

- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

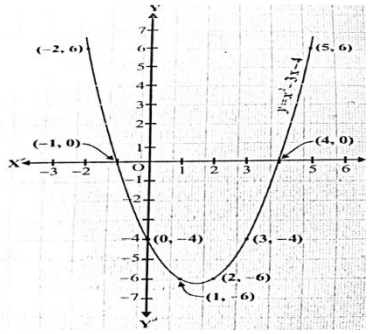
Graphs of different types of polynomials:

- Linear polynomial:- The graph of a linear polynomial ax+b is a straight line, intersecting X- axis at one point

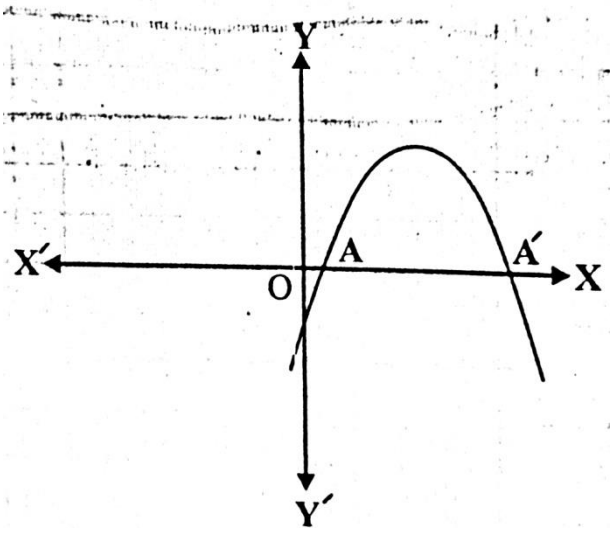


- Quadratic polynomial:-

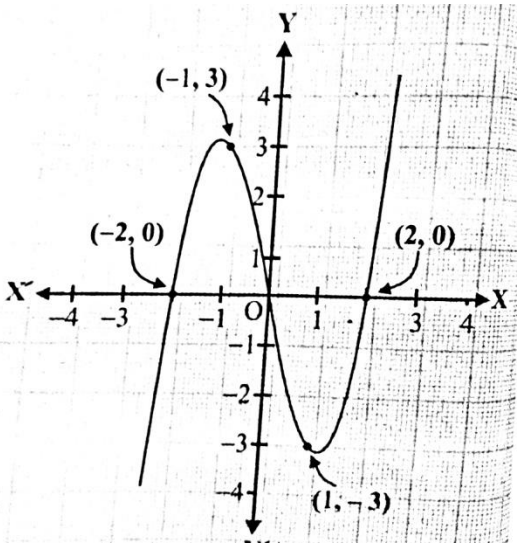
  (i) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a>0 & intersects x-axis at maximum two distinct points.



(ii) Graph of a quadratic polynomial p(x)=$ax^2 + bx + c$ is a parabola open downwards like ∩ if a<0 & intersects x-axis at maximum two distinct points



- Cubic polynomial and its graph:- in general a polynomial p(x) of degree n crosses the x-axis at most n points.



<u>For a quadratic polynomial</u>:- If α, β are zeroes of $p(x) = ax^2 + bx + c$ then,

1. Sum of zeroes $= \alpha + \beta = -\frac{b}{a} = \frac{-coefficients\ of\ x}{coefficient\ of\ x^2}$
2. Product of zeroes= $\alpha.\beta = \frac{c}{a} = \frac{constant\ term}{coefficient\ of x^2}$

- A quadratic polynomial whose zeroes are $\alpha\ and\ \beta$, is given by:

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

- If $\alpha, \beta\ and\ \gamma$ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

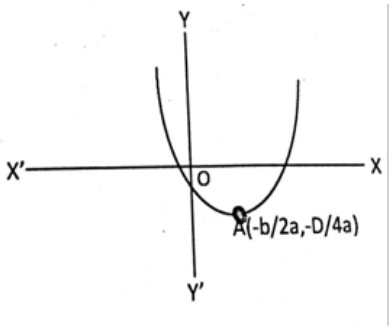
- If $\alpha, \beta \& \gamma$ are zeroes of a cubic polynomial p(x),

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

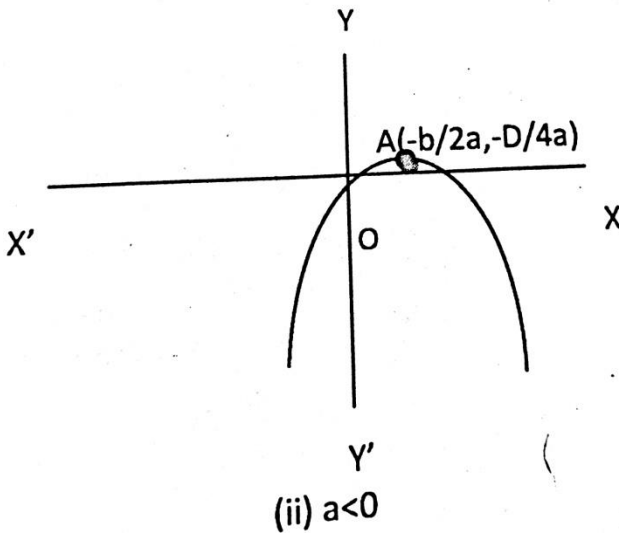
Division algorithm for polynomials: If p(x) and g(x) are any two polynomials with g(x)$\neq 0$, then we have polynomials q(x) and r(x) such that
P(x) = g(x)$\times q(x) + r(x)$, where r(x) =0 or degree of r(x) <degree of g(x).

Nature of graph of polynomial$P(x) = ax^2 + bx + c$:-
Case-1 When polynomial $ax^2 + bx + c$ is factorable in two distinct linear factors.

In this case, curve cuts X- axis at two distinct points. The co-ordinate of the vertex of parabola are(-b/2a, -D/2a) where D=$b^2 - 4ac$. The x co-ordinates of these points are the two zeroes of the polynomial.



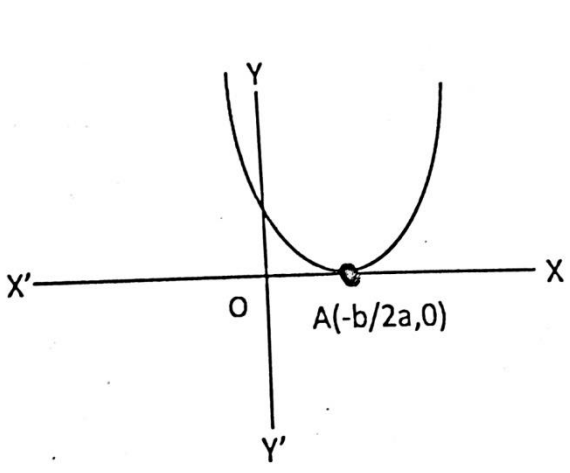
(i) a>0

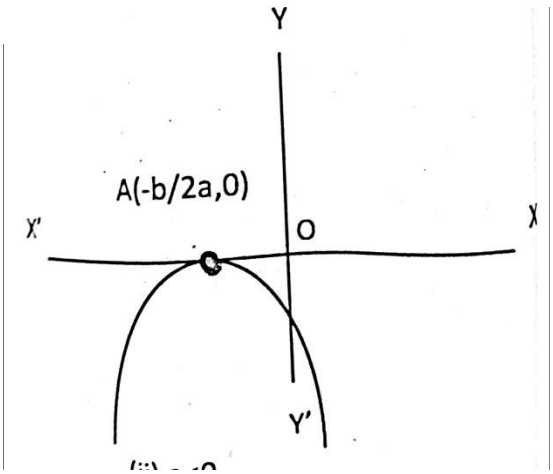


(ii) a<0

Case 2:- When Polynomial $ax^2 + bx + c$ is factorisable into two equal factors.

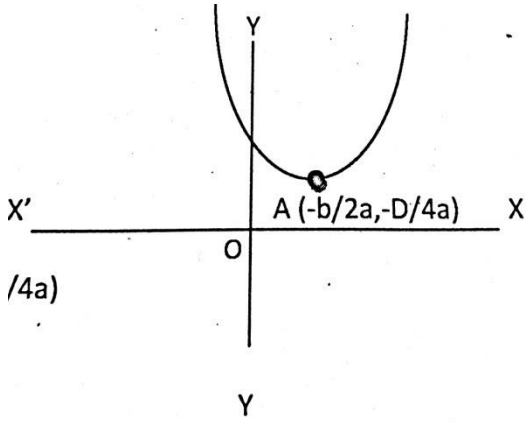
In this case, curve touches X-axis at the point (-b/2a, 0). The x- Co-ordinates of the point gives two equal zeroes of the polynomial.

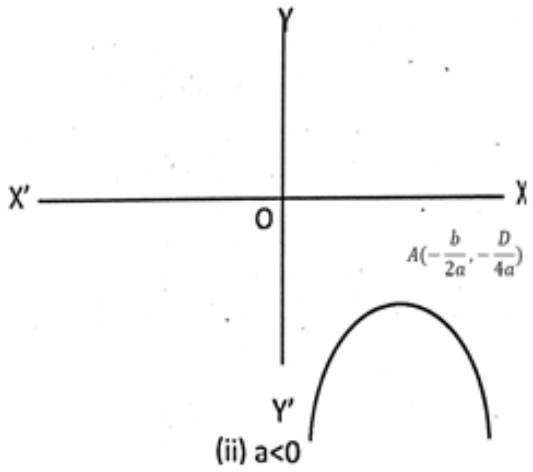




(i) a>0 (ii) a<0

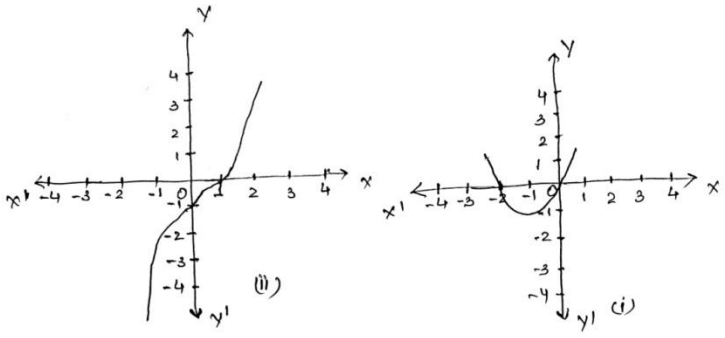
Case- 3 When Polynomial $ax^2 + bx + c$ is not factorizable. In this case, the curve doesn't cut or touches X-axis





Level –I

1. Find the value of zeroes of the polynomials p(x) as shown in the graph and hence find the polynomial.(CBSE 2014-15).



2. Let α and β are the zeroes of a quadratic polynomial $2x^2 - 5x - 6$ then form a quadratic polynomial whose zeroes are $\alpha + \beta \; and \; \alpha\beta$. (CBSE 2011)

3. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$? (CBSE 2010)
4. Can (x-7) be the remainder on division of a polynomial $p(x)\ by\ (7x + 2)$? Justify your answer(CBSE 2010)
5. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$? (CBSE 2013)
6. Write the degree of zero polynomial?
7. Find the zeroes of a quadratic polynomial $6x^2 - 7x - 3$ and verify the relationship between the zeroes and the coefficients? (CBSE 2014-15
8. Find the quadratic polynomial sum of whose zeroes is 2√3 and their product is 2?(CBSE 2008)

Level II

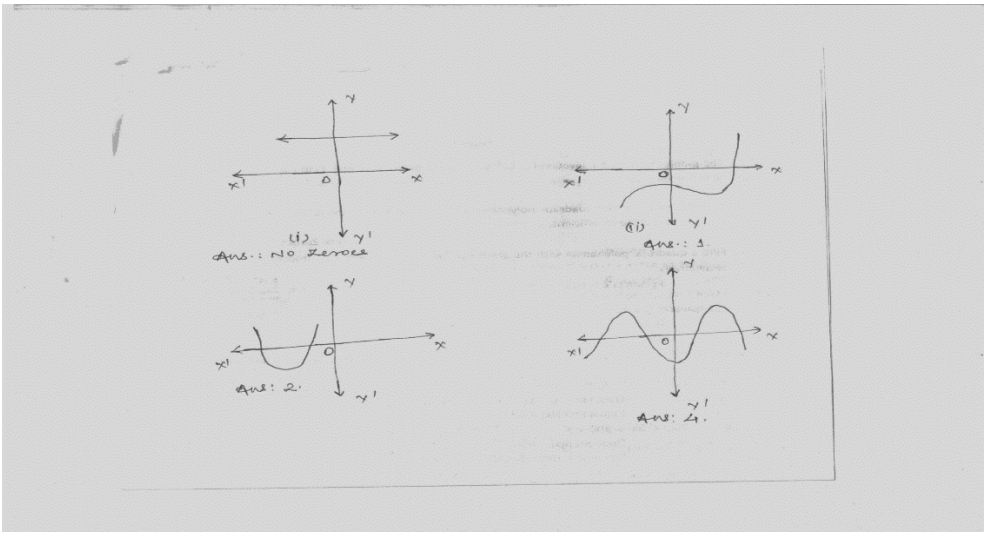
9. If the sum of squares of the zeroes of the polynomials $6x^2 + x + k$ is $\frac{25}{36}$. find the value of k?(CBSE 2014-15)
10. If one zero of the quadratic polynomial f(x)= $4x^2 - 8kx - 9$ is negative of the other, then find the value of k?(CBSE 2014-15)
11. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots. (CBSE 2014)
12. On dividing $3x^3 - 2x^2 + 5x + 5$ by the polynomial p(x), the quotient and remainder are $x^2 - x + 2$ and-7 respectively. Find p(x)?(CBSE 2013)
13. Find all the zeroes of the polynomial $x^4 + x^3 - 9x^2 - 3x + 18$, if two of its zeroes are $\sqrt{3}\ and\sqrt{-3}$. (CBSE 2010,13)
14. If α , β are zeroes of the quadratic polynomial $p(x) = x^2 - (k - 6)x + (2k + 1)$. Find the value of k if $\alpha + \beta = \alpha\beta$. (CBSE 2010)
15. If the zeroes of the polynomial $x^2 - 5x + k$ are the reciprocal of each other, then find the value of K? (CBSE 2011)
16. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$, find the value of$'a'$. If $3\alpha + 2\beta = 20$.(CBSE 2010)

LEVEL III

17. On dividing $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and remainder are $3x + 10$ and $16x - 43$ respectively. Find the polynomial g(x). (CBSE 14-15)
18. If -5 is a root of quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x)k = 0$ has equal roots, find the value of k. (CBSE 2106)
19. If $\alpha, \beta \; and \; \gamma$ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the values of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. (CBSE 2010)
20. Form a cubic polynomial whose zeroes are 3, 2 and -1. Hence find
    (i) Sum of its zeroes
    (ii) Sum of the product, taken two at a time
    (iii) Product of its zero.

(SELF EVALUATION QUESTIONS)

21. Find the number of zeroes of p(x) in each case, for some polynomials p(x).



22. If $\alpha \; and \beta$ are the zeroes of the equation $6x^2 + x - 2 = 0$, find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
23. If one of the zeroes of the polynomial $2x^2 + px + 4 = 0$ is 2, find the other zero, also find the value of p
24. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a. (All India)

Value Based Questions

25. If α be the number of person who take junk food, β be the person who take food at home and α and β be the zeroes of quadratic polynomial $f(x) = x^2 - 3x + 2$, then find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$, which way of taking food you prefer and why?
26. If the number of apples and mangoes are the zeroes of the polynomial $3x^2 = 8x - 2k + 1$ and the number of apples is 7 times the number of mangoes, then find the number of zeroes and value of k. What are benefits of fruits in our daily life?