

POLYNOMIALS

It is not once nor twice but times without number that the same ideas make their appearance in the world.

1. Find the value for K for which $x^4 + 10x^3 + 25x^2 + 15x + K$ exactly divisible by $x + 7$.

(Ans : K= - 91)

Ans: Let $P(x) = x^4 + 10x^3 + 25x^2 + 15x + K$ and $g(x) = x + 7$

Since $P(x)$ exactly divisible by $g(x)$

$$\therefore r(x) = 0$$

$$\begin{array}{r} \text{now } x + 7 \overline{) x^4 + 10x^3 + 25x^2 + 15x + K} \\ \underline{x^4 + 7x^3} \\ 3x^3 + 25x^2 \\ \underline{3x^3 + 21x^2} \\ 4x^2 + 15x \\ \underline{4x^2 + 28x} \\ -13x + K \\ \underline{-13x - 91} \\ K + 91 \\ \hline \end{array}$$

$$\therefore K + 91 = 0$$

K= -91

2. If two zeros of the polynomial $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$. Find the other zeros.
(Ans: 7, -5)

Ans: Let the two zeros are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

$$\begin{aligned} \text{Sum of Zeros} &= 2 + \sqrt{3} + 2 - \sqrt{3} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{Product of Zeros} &= (2 + \sqrt{3})(2 - \sqrt{3}) \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

Quadratic polynomial is $x^2 - (\text{sum})x + \text{Product}$

$$\begin{array}{r}
 x^2 - 2x - 35 \\
 x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{x^4 - 4x^3 + x^2} \\
 -2x^3 - 27x^2 + 138x \\
 \underline{-2x^3 + 8x^2 - 2x} \\
 -35x^2 + 140x - 35 \\
 \underline{-35x^2 + 140x - 35} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore x^2 - 2x - 35 &= 0 \\
 (x - 7)(x + 5) &= 0 \\
 x &= 7, -5
 \end{aligned}$$

other two Zeros are 7 and -5

3. Find the Quadratic polynomial whose sum and product of zeros are $\sqrt{2} + 1, \frac{1}{\sqrt{2} + 1}$.

Ans: sum = $2\sqrt{2}$
Product = 1
Q.P =
 $X^2 - (\text{sum})x + \text{Product}$

$$\therefore x^2 - (2\sqrt{2})x + 1$$

4. If α, β are the zeros of the polynomial $2x^2 - 4x + 5$ find the value of a) $\alpha^2 + \beta^2$ b) $(\alpha - \beta)^2$.

(Ans: a) -1 , b) -6)

Ans: $p(x) = 2x^2 - 4x + 5$

$$\alpha + \beta = \frac{-b}{a} = \frac{4}{2} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Substitute then we get, $\alpha^2 + \beta^2 = -1$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

Substitute, we get $(\alpha - \beta)^2 = -6$

5. If α, β are the zeros of the polynomial $x^2 + 8x + 6$ form a Quadratic polynomial

whose zeros are a) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ b) $1 + \frac{\beta}{\alpha}$, $1 + \frac{\alpha}{\beta}$.

$$(\text{Ans: } x^2 + \frac{4}{3}x + \frac{1}{6}, x^2 - \frac{32}{3}x + \frac{32}{3})$$

Ans: $p(x) = x^2 + 8x + 6$
 $\alpha + \beta = -8$ and $\alpha\beta = 6$

a) Let two zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

$$\text{Sum} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-8}{6} = \frac{-4}{3}$$

$$\text{Product} = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{6}$$

Required Q.P is

$$x^2 + \frac{4}{3}x + \frac{1}{6}$$

b) Let two Zeros are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$

$$\text{sum} = 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$$

$$= 2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= 2 + \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= 2 + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \text{ after solving this problem,}$$

$$\text{We get} = \frac{32}{3}$$

$$\text{Product} = \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$$

$$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1$$

$$= 2 + \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

Substitute this sum,

$$\text{We get} = \frac{32}{3}$$

$$\text{Required Q.P. is } x^2 - \frac{32}{3}x + \frac{32}{3}$$

6. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$ the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find the polynomial $g(x)$.
(Ans: $4x^2 + 7x + 2$)

Ans: $p(x) = g(x)q(x) + r(x)$

$$g(x) = \frac{p(x) - r(x)}{q(x)}$$

$$\text{let } p(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$$

$$q(x) = x^2 - 3x - 5 \text{ and } r(x) = -5x + 8$$

$$\text{now } p(x) - r(x) = 4x^4 - 5x^3 - 39x^2 - 41x - 10$$

$$\text{when } \frac{p(x) - r(x)}{q(x)} = 4x^2 + 7x + 2$$

$$\therefore g(x) = 4x^2 + 7x + 2$$

7. If the squared difference of the zeros of the quadratic polynomial $x^2 + px + 45$ is equal to 144, find the value of p . (Ans: ± 18).

Ans: Let two zeros are α and β where $\alpha > \beta$

According given condition

$$(\alpha - \beta)^2 = 144$$

$$\text{Let } p(x) = x^2 + px + 45$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{45}{1} = 45$$

$$\text{now } (\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$\text{Solving this we get } p = \pm 18$$

8. If α, β are the zeros of a Quadratic polynomial such that $\alpha + \beta = 24$, $\alpha - \beta = 8$. Find a Quadratic polynomial having α and β as its zeros. (Ans: $k(x^2 - 24x + 128)$)

Ans: $\alpha + \beta = 24$

$$\alpha - \beta = 8$$

$$2\alpha = 32$$

$$\alpha = \frac{32}{2} = 16, \therefore \alpha = 16$$

Work the same way to $\alpha + \beta = 24$

$$\text{So, } \beta = 8$$

Q.P is $x^2 - (\text{sum})x + \text{product}$
 $= x^2 - (16+8)x + 16 \times 8$
 Solve this,
 it is $k(x^2 - 24x + 128)$

9. If α & β are the zeroes of the polynomial $2x^2 - 4x + 5$, then find the value of

a. $\alpha^2 + \beta^2$ b. $1/\alpha + 1/\beta$ c. $(\alpha - \beta)^2$ d. $1/\alpha^2 + 1/\beta^2$ e. $\alpha^3 + \beta^3$

$$(\text{Ans: } -1, \frac{4}{5}, -6, \frac{-4}{25}, -7)$$

Ans: Let $p(x) = 2x^2 - 4x + 5$

$$\alpha + \beta = \frac{-b}{a} = \frac{4}{2} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Substitute to get $\alpha^2 + \beta^2 = -1$

b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

substitute, then we get $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{4}{5}$

b) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Therefore we get, $(\alpha - \beta)^2 = -6$

d) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha\beta^2} = \frac{-1}{\left(\frac{5}{2}\right)^2}$

$$\therefore \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{-4}{25}$$

e) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

Substitute this,

to get, $\alpha^3 + \beta^3 = -7$

10. Obtain all the zeros of the polynomial $p(x) = 3x^4 - 15x^3 + 17x^2 + 5x - 6$ if two zeroes are $-1/\sqrt{3}$ and $1/\sqrt{3}$. (Ans:3,2)
11. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$ which satisfy the division algorithm.
a. $\deg p(x) = \deg q(x)$ b. $\deg q(x) = \deg r(x)$ c. $\deg q(x) = 0$.
12. If the ratios of the polynomial $ax^3 + 3bx^2 + 3cx + d$ are in AP, Prove that $2b^3 - 3abc + a^2d = 0$

Ans: Let $p(x) = ax^3 + 3bx^2 + 3cx + d$ and α, β, r are their three Zeros

but zero are in AP

let $\alpha = m - n$, $\beta = m$, $r = m + n$

$$\text{sum} = \alpha + \beta + r = \frac{-b}{a}$$

$$\text{substitute this sum, to get } m = \frac{-b}{a}$$

$$\text{Now taking two zeros as sum } \alpha\beta + \beta r + \alpha r = \frac{c}{a}$$

$$(m-n)m + m(m+n) + (m+n)(m-n) = \frac{3c}{a}$$

Solve this problem, then we get

$$\frac{3b^2 - 3ac}{a^2} = n^2$$

$$\text{Product } \alpha\beta r = \frac{d}{a}$$

$$(m-n)m(m+n) = \frac{-d}{a}$$

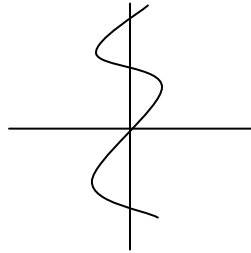
$$(m^2 - n^2)m = \frac{-d}{a}$$

$$\left[\left(\frac{-b}{a}\right)^2 - \left(\frac{3b^2 - 3ac}{a^2}\right)\right] \left(\frac{-b}{a}\right) = \frac{-d}{a}$$

Simplifying we get

$$2b^3 - 3abc + a^2d = 0$$

13. Find the number of zeros of the polynomial from the graph given.



(Ans:1)

14. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the zeros and the value of k (Ans $k = 2/3$)

Self Practice

14. If $(n-k)$ is a factor of the polynomials $x^2 + px + q$ & $x^2 + mx + n$. Prove that

$$k = n + \frac{n-q}{m-p}$$

Ans : since $(n-k)$ is a factor of $x^2 + px + q$

$$\therefore (n-k)^2 + p(n-k) + q = 0$$

$$\text{And } (n-k)^2 + m(n-k) + n = 0$$

Solve this problem by yourself,

$$\therefore k = n + \frac{n-q}{m-p}$$

SELF PRACTICE

16. If $2, \frac{1}{2}$ are the zeros of $px^2 + 5x + r$, prove that $p = r$.
 17. If m, n are zeroes of $ax^2 - 5x + c$, find the value of a and c if $m + n = m \cdot n = 10$ (Ans: $a = 1/2, c = 5$)
 18. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$. (Ans: $14x - 10$)
 19. What must be added to the polynomial $p(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$. (Ans: $x - 2$)