

6. Trigonometric Identities

Exercise 6.1

1. Question

Prove the following trigonometric identities:

$$(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$$

Answer

$$\text{To Prove: } (1 - \cos^2 A) \operatorname{cosec}^2 A = 1$$

Proof:

$$1 - \cos^2 A = \sin^2 A$$

$$\text{Therefore, } L.H.S = \sin^2 A \cdot \operatorname{cosec}^2 A$$

$$\text{Now, } \operatorname{cosec}^2 A = \frac{1}{\sin^2 A}$$

$$\text{Therefore, } L.H.S = \sin^2 A \cdot \frac{1}{\sin^2 A} = 1$$

$$R.H.S = 1$$

$$L.H.S = R.H.S$$

Hence, Proved

2. Question

Prove the following trigonometric identities:

$$(1 + \cot^2 A) \sin^2 A = 1$$

Answer

Consider,

$$(1 + \cot^2 A) \sin^2 A$$

$$\text{As we know } 1 + \cot^2 A = \operatorname{cosec}^2 A$$

Putting the values we get,

$$(\operatorname{cosec}^2 A) \sin^2 A$$

$$\text{As we know, } \operatorname{cosec} A = 1/\sin A$$

So,

$$\Rightarrow \frac{1}{\sin^2 A} \times \sin^2 A = 1$$

hence proved

3. Question

Prove the following trigonometric identities:

$$\tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$$

Answer

$$\begin{aligned}\tan^2 \theta \cos^2 \theta &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \cos^2 \theta \\ &= \sin^2 \theta \\ &= 1 - \cos^2 \theta\end{aligned}$$

Hence Proved.

4. Question

Prove the following trigonometric identities:

$$\operatorname{cosec} \theta \sqrt{1 - \cos^2 \theta} = 1$$

Answer

$$\begin{aligned}\operatorname{cosec} \theta \sqrt{1 - \cos^2 \theta} &= \frac{1}{\sin \theta} \sqrt{\sin^2 \theta} \\ &= \frac{1}{\sin \theta} \times \sin \theta \\ &= 1\end{aligned}$$

Hence Proved.

5. Question

Prove the following trigonometric identities:

$$(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) = 1$$

Answer

$$\begin{aligned}(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) &= \tan^2 \theta \times \cot^2 \theta \\ &= 1\end{aligned}$$

Hence Proved.

6. Question

Prove the following trigonometric identities:

$$\tan \theta + \frac{1}{\tan \theta} = \sec \theta \operatorname{cosec} \theta$$

Answer

$$\begin{aligned}\tan \theta + \frac{1}{\tan \theta} &= \frac{\tan^2 \theta + 1}{\tan \theta} \\ &= \sec^2 \theta \times \frac{\cos \theta}{\sin \theta} \\ &= \frac{1}{\cos^2 \theta} \times \frac{\cos \theta}{\sin \theta} \\ &= \frac{1}{\cos \theta} \times \frac{1}{\sin \theta} \\ &= \sec \theta \operatorname{cosec} \theta\end{aligned}$$

Hence Proved.

7. Question

Prove the following trigonometric identities:

$$\frac{\cos \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$$

Answer

$$\begin{aligned}\frac{\cos \theta}{1 - \sin \theta} &= \frac{\cos \theta}{1 - \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta} \\ &= \frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta} \\ &= \frac{\cos \theta (1 + \sin \theta)}{\cos^2 \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta}\end{aligned}$$

Hence Proved.

8. Question

Prove the following trigonometric identities:

$$\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

Answer

$$\begin{aligned}\frac{\cos \theta}{1 + \sin \theta} &= \frac{\cos \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \\&= \frac{\cos \theta (1 - \sin \theta)}{1 - \sin^2 \theta} \\&= \frac{\cos \theta (1 - \sin \theta)}{\cos^2 \theta} \\&= \frac{1 - \sin \theta}{\cos \theta}\end{aligned}$$

Hence Proved.

9. Question

Prove the following trigonometric identities:

$$\cos^2 A + \frac{1}{1 + \cot^2 A} = 1$$

Answer

$$\begin{aligned}\cos^2 A + \frac{1}{1 + \cot^2 A} &= \cos^2 A + \frac{1}{\operatorname{cosec}^2 A} \\&= \cos^2 A + \sin^2 A \\&= 1\end{aligned}$$

Hence Proved.

10. Question

Prove the following trigonometric identities:

$$\sin^2 A + \frac{1}{1 + \tan^2 A} = 1$$

Answer

$$\begin{aligned}\sin^2 A + \frac{1}{1 + \tan^2 A} &= \sin^2 A + \frac{1}{\sec^2 A} \\&= \sin^2 A + \cos^2 A \\&= 1\end{aligned}$$

Hence Proved.

11. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta$$

Answer

$$\begin{aligned}\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} &= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}} \\&= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} \\&= \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}} \\&= \frac{1 - \cos \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \\&= \operatorname{cosec} \theta - \cot \theta\end{aligned}$$

Hence Proved.

12. Question

Prove the following trigonometric identities:

$$\frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

Answer

$$\begin{aligned}\frac{1 - \cos \theta}{\sin \theta} &= \frac{1 - \cos \theta}{\sin \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\&= \frac{1 - \cos^2 \theta}{\sin \theta (1 + \cos \theta)} \\&= \frac{\sin^2 \theta}{\sin \theta (1 + \cos \theta)} \\&= \frac{\sin \theta}{1 + \cos \theta}\end{aligned}$$

Hence Proved.

13. Question

Prove the following trigonometric identities:

$$\frac{\sin \theta}{1 - \cos \theta} = \operatorname{cosec} \theta + \cot \theta$$

Answer

$$\begin{aligned}\frac{\sin \theta}{1 - \cos \theta} &= \frac{\sin \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\&= \frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} \\&= \frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta} \\&= \frac{1 + \cos \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \\&= \operatorname{cosec} \theta + \cot \theta\end{aligned}$$

$$\text{As, } \frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

$$\text{and } \frac{\cos \theta}{\sin \theta} = \cot \theta$$

Hence Proved.

14. Question

Prove the following trigonometric identities:

$$\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$$

Answer

$$\begin{aligned}\frac{1 - \sin \theta}{1 + \sin \theta} &= \frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \\&= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta} \\&= \frac{(1 - \sin \theta)^2}{\cos^2 \theta} \\&= \left(\frac{1 - \sin \theta}{\cos \theta} \right)^2 \\&= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2 \\&= (\sec \theta - \tan \theta)^2\end{aligned}$$

Hence Proved.

15. Question

Prove the following trigonometric identities:

$$(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta) = \cot^2 \theta + \cos^2 \theta$$

Answer

Consider,

$$(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta)$$

Apply the formula $(a^2 - b^2) = (a+b)(a-b)$

we get,

$$(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta) = \operatorname{cosec}^2 \theta - \sin^2 \theta$$

As we know $1 + \cot^2 A = \operatorname{cosec}^2 A$

and $1 - \cos^2 A = \sin^2 A$

So,

$$(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta) = (1 + \cot^2 A) - (1 - \cos^2 A)$$

$$= 1 + \cot^2 A - 1 + \cos^2 A$$

$$= \cot^2 A + \cos^2 A$$

Hence Proved.

16. Question

Prove the following trigonometric identities:

$$\frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} = \cot \theta$$

Answer

To prove: $\frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} = \cot \theta$

Proof: Use the identity $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ and the formula $\cos \theta = 1 / \sec \theta$ and $\operatorname{cosec} \theta = 1 / \sin \theta$, $\tan \theta = \sin \theta / \cos \theta$

$$\begin{aligned} \frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} &= \frac{\operatorname{cosec}^2 \theta \times \tan \theta}{\sec^2 \theta} \\ &= \frac{\cos^2 \theta \times \tan \theta}{\sin^2 \theta} \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} \times \tan \theta \\ &= \cot^2 \theta \times \tan \theta \\ &= \cot^2 \theta \times \frac{1}{\cot \theta} \\ &= \cot \theta \end{aligned}$$

Hence Proved.

17. Question

Prove the following trigonometric identities:

$$(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta$$

Answer

To Prove: $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta$

Proof: Use the formula: $(a + b)(a - b) = a^2 - b^2$ on $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta)$

Where $a = \sec \theta$ and $b = \cos \theta$

so,

$$(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \sec^2 \theta - \cos^2 \theta \dots\dots (1)$$

We know, $\sec^2 \theta = \tan^2 \theta + 1$

$$\sin^2\theta + \cos^2\theta = 1$$

Use the identities in the eq. (1) $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \sec^2\theta - \cos^2\theta$

$$= (\tan^2\theta + 1) - (1 - \sin^2\theta)$$

$$= \tan^2\theta + 1 - 1 + \sin^2\theta$$

$$= \tan^2\theta + \sin^2\theta \text{ Hence proved.}$$

18. Question

Prove the following trigonometric identities:

$$\sec A(1 - \sin A)(\sec A + \tan A) = 1$$

Answer

$$\begin{aligned} & \sec A(1 - \sin A)(\sec A + \tan A) \\ &= \left(\sec A - \frac{1}{\cos A} \times \sin A \right) (\sec A + \tan A) \\ &= (\sec A - \tan A)(\sec A + \tan A) \\ &= (\sec^2 A - \tan^2 A) \\ &= (\tan^2 A + 1 - \tan^2 A) \\ &= 1 \end{aligned}$$

Hence Proved.

19. Question

Prove the following trigonometric identities:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$$

Answer

taking LHS

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$ As we know, $\operatorname{cosec} A = 1/\sin A$, $\sec A = 1/\cos A$, $\tan A = \sin / \cos A$ So,

$$\begin{aligned} &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \\ &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \left(\frac{\sin^2 A + \cos^2 A}{\cos A} \right) \end{aligned}$$

As we know, $\sin^2 A + \cos^2 A = 1$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \left(\frac{1}{\cos A} \right) = 1$$

Hence Proved.

20. Question

Prove the following trigonometric identities:

$$\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

Answer

$$\begin{aligned}
 \text{LHS. : } \tan^2 \theta - \sin^2 \theta &= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta \\
 &= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta - \sin^2 \theta (1 - \sin^2 \theta)}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta - \sin^2 \theta + \sin^4 \theta}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \sin^2 \theta \\
 &= \tan^2 \theta \sin^2 \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

Hence Proved.

21. Question

Prove the following trigonometric identities:

$$(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) = 1$$

Answer

$$\begin{aligned}
 &(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) \\
 &= \sec^2 \theta (1 - \sin^2 \theta) \\
 &= \sec^2 \theta \cos^2 \theta \\
 &= 1
 \end{aligned}$$

Hence Proved.

22. Question

Prove the following trigonometric identities:

$$\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$$

Answer

$$\text{given : } \sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$$

To prove : Above equality holds.

Proof: Consider LHS, we know,

$$\cot \theta = \frac{\cos \theta}{\sin \theta} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

using these

$$\begin{aligned}
 &\sin^2 A \cot^2 A + \cos^2 A \tan^2 A \\
 &= \sin^2 A \times \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \times \frac{\sin^2 A}{\cos^2 A} \\
 &= \cos^2 A + \sin^2 A \\
 &= 1
 \end{aligned}$$

Which is equal to RHS.

Hence Proved.

23 A. Question

Prove the following trigonometric identities:

$$\cot \theta - \tan \theta = \frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta}$$

Answer

$$\begin{aligned}
 L.H.S : \cot \theta - \tan \theta &= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cdot \cos \theta} \\
 &= \frac{\cos^2 \theta - (1 - \cos^2 \theta)}{\sin \theta \cdot \cos \theta} \\
 &= \frac{\cos^2 \theta - 1 + \cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
 &= \frac{2\cos^2 \theta - 1}{\sin \theta \cdot \cos \theta} = R.H.S
 \end{aligned}$$

Hence Proved.

23 B. Question

Prove the following trigonometric identities:

$$\tan \theta - \cot \theta = \frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta}$$

Answer

$$\begin{aligned}
 L.H.S : \tan \theta - \cot \theta &= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta - (1 - \sin^2 \theta)}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta - 1 + \sin^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta}
 \end{aligned}$$

Hence Proved.

24. Question

Prove the following trigonometric identities:

$$\frac{\cos^2 \theta}{\sin \theta} - \operatorname{cosec} \theta + \sin \theta = 0$$

Answer

$$\begin{aligned}
 \frac{\cos^2 \theta}{\sin \theta} - \operatorname{cosec} \theta + \sin \theta &= \frac{\cos^2 \theta}{\sin \theta} - \frac{1}{\sin \theta} + \sin \theta \\
 &= \frac{\cos^2 \theta - 1 + \sin^2 \theta}{\sin \theta} \\
 &= \frac{(\cos^2 \theta + \sin^2 \theta) - 1}{\sin \theta} \\
 &= \frac{1 - 1}{\sin \theta} = 0
 \end{aligned}$$

Hence Proved.

25. Question

Prove the following trigonometric identities:

$$\frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = 2 \sec^2 A$$

Answer

$$\begin{aligned}
 \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} &= \frac{1 - \sin A + 1 + \sin A}{1 - \sin^2 A} \\
 &= \frac{2}{\cos^2 A} \\
 &= 2 \sec^2 A
 \end{aligned}$$

Hence Proved.

26. Question

Prove the following trigonometric identities:

$$\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$$

Answer

$$\begin{aligned} &= \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{1 + \sin^2 \theta + 2 \sin \theta + \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{1 + 1 + 2 \sin \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{2 + 2 \sin \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{2(1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{2}{\cos \theta} \\ &= 2 \sec \theta \end{aligned}$$

Hence Proved.

27. Question

Prove the following trigonometric identities:

$$\frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$$

Answer

$$\begin{aligned} &\frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \\ &= \frac{1 + \sin^2 \theta + 2 \sin \theta + 1 + \sin^2 \theta - 2 \sin \theta}{2 \cos^2 \theta} \\ &= \frac{2 + 2 \sin^2 \theta}{2 \cos^2 \theta} \\ &= \frac{2(1 + \sin^2 \theta)}{2 \cos^2 \theta} \\ &= \frac{(1 + \sin^2 \theta)}{\cos^2 \theta} \\ &= \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \end{aligned}$$

Hence Proved.

28. Question

Prove the following trigonometric identities:

$$\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \left(\frac{1 - \tan \theta}{1 - \cot \theta} \right)^2 = \tan^2 \theta$$

Answer

Use the formula:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\cos^2 \theta}{\sin^2 \theta}} = \frac{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

Now,

$$\begin{aligned} \left(\frac{1 - \tan \theta}{1 - \cot \theta} \right)^2 &= \left(\frac{1 - \frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} \right)^2 = \left(\frac{\frac{\cos \theta - \sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} \right)^2 \\ &= \left(\frac{\cos \theta - \sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta - \cos \theta} \right)^2 = \left(\frac{\cos \theta - \sin \theta}{\cos \theta} \times \frac{\sin \theta}{-(\cos \theta - \sin \theta)} \right)^2 \\ &= \left(\frac{\sin \theta}{\cos \theta} \right)^2 \\ &= \tan^2 \theta \end{aligned}$$

Hence Proved.

29. Question

Prove the following trigonometric identities:

$$\frac{1 + \sec \theta}{\sec \theta} = \frac{\sin^2 \theta}{1 - \cos \theta}$$

Answer

$$\begin{aligned} \frac{1 + \sec \theta}{\sec \theta} &= \frac{1 + \frac{1}{\cos \theta}}{\frac{1}{\cos \theta}} \\ &= \frac{1 + \cos \theta}{1} \\ &= \frac{(1 + \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)} \\ &= \frac{1 - \cos^2 \theta}{(1 - \cos \theta)} \\ &= \frac{\sin^2 \theta}{1 - \cos \theta} \end{aligned}$$

Hence Proved.

30. Question

Prove the following trigonometric identities:

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$$

Answer

Given : $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$

To prove: Above equality.

Taking LHS Use $\tan \theta = \frac{1}{\cot \theta}$, $\cot \theta = \frac{1}{\tan \theta}$

$$\begin{aligned}
& \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
&= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\frac{1}{\tan \theta}}{1 - \tan \theta} \\
&= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{1}{\tan \theta (1 - \tan \theta)} \\
&= \frac{1}{(\tan \theta - 1)} \left(\tan^2 \theta - \frac{1}{\tan \theta} \right) \\
&= \frac{\tan^3 \theta - 1}{\tan \theta (\tan \theta - 1)} \\
&= \frac{(\tan \theta - 1)(\tan^2 \theta + \tan \theta + 1)}{\tan \theta (\tan \theta - 1)}
\end{aligned}$$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$\begin{aligned}
&= \frac{\tan^2 \theta + \tan \theta + 1}{\tan \theta} \\
&= \tan \theta + 1 + \cot \theta
\end{aligned}$$

= RHS Hence Proved.

31. Question

Prove the following trigonometric identities:

$$\sec^6 \theta = \tan^6 \theta + 3 \tan^2 \theta \sec^2 \theta + 1$$

Answer

Taking RHS

$$\tan^6 \theta + 3 \tan^2 \theta \sec^2 \theta + 1$$

$$= (\sec^2 \theta - 1)^3 + 3(\sec^2 \theta - 1)\sec^2 \theta + 1$$

$$[\text{As, } \tan^2 \theta = \sec^2 \theta - 1]$$

$$= (\sec^6 \theta - 1 - 3\sec^4 \theta + 3\sec^2 \theta) + (3\sec^4 \theta - 3\sec^2 \theta) + 1$$

$$[(a + b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$$

$$= \sec^6 \theta = \text{LHS Hence Proved.}$$

32. Question

Prove the following trigonometric identities:

$$\operatorname{cosec}^6 \theta = \cot^6 \theta + 3 \cot^2 \theta \operatorname{cosec}^2 \theta + 1$$

Answer

$$\operatorname{cosec}^3 \theta = \cot^3 \theta + 3 \cot^2 \theta \operatorname{cosec}^2 \theta + 1$$

$$\operatorname{cosec}^3 \theta - \cot^3 \theta - 3 \cot^2 \theta \operatorname{cosec}^2 \theta = 1 \quad \dots (i)$$

since we know that

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

so we can write LHS of eq. (i) as

$$(\operatorname{cosec}^2 \theta)^3 - (\cot^2 \theta)^3 - 3 \cot^2 \theta \operatorname{cosec}^2 \theta (\operatorname{cosec}^2 \theta - \cot^2 \theta) \quad \dots (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1)$$

$$= (\operatorname{cosec}^2 \theta - \cot^2 \theta)^3$$

$$= 1 = \text{RHS}$$

Hence Proved

33. Question

Prove the following trigonometric identities:

$$\frac{(1 + \tan^2 \theta) \cot \theta}{\operatorname{cosec}^2 \theta} = \tan \theta$$

Answer

$$\begin{aligned} \frac{(1 + \tan^2 \theta) \cot \theta}{\operatorname{cosec}^2 \theta} &= \frac{\sec^2 \theta \times \cot \theta}{\operatorname{cosec}^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \cot \theta \\ &= \tan^2 \theta \times \cot \theta \\ &= \tan^2 \theta \times \frac{1}{\tan \theta} \\ &= \tan \theta \end{aligned}$$

Hence Proved.

34. Question

Prove the following trigonometric identities:

$$\frac{1 + \cos A}{\sin^2 A} = \frac{1}{1 - \cos A}$$

Answer

$$\begin{aligned} \frac{1 + \cos A}{\sin^2 A} &= \frac{1 + \cos A}{\sin^2 A} \times \frac{1 - \cos A}{1 - \cos A} \\ &= \frac{1 - \cos^2 A}{\sin^2 A (1 - \cos A)} \\ &= \frac{\sin^2 A}{\sin^2 A (1 - \cos A)} \\ &= \frac{1}{1 - \cos A} \end{aligned}$$

Hence Proved.

35. Question

Prove the following trigonometric identities:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$

Answer

$$\begin{aligned} \text{R.H.S: } \frac{\cos^2 A}{(1 + \sin A)^2} &= \frac{1 - \sin^2 A}{(1 + \sin A)^2} \\ &= \frac{(1 - \sin A)(1 + \sin A)}{(1 + \sin A)^2} \\ &= \frac{(1 - \sin A) / \cos A}{(1 + \sin A) / \cos A} \\ &= \frac{\left(\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right)}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)} \\ &= \frac{\sec A - \tan A}{\sec A + \tan A} = \text{L.H.S} \end{aligned}$$

Hence Proved.

36. Question

Prove the following trigonometric identities:

$$\frac{1 + \cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

Answer

$$\begin{aligned}\frac{1 + \cos A}{\sin A} &= \frac{1 + \cos A}{\sin A} \times \frac{1 - \cos A}{1 - \cos A} \\ &= \frac{1 - \cos^2 A}{\sin A (1 - \cos A)} \\ &= \frac{\sin^2 A}{\sin A (1 - \cos A)} \\ &= \frac{\sin A}{1 - \cos A}\end{aligned}$$

Hence Proved.

37. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Answer

$$\begin{aligned}\sqrt{\frac{1 + \sin A}{1 - \sin A}} &= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\ &= \frac{1 + \sin A}{\cos A} \\ &= \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\ &= \sec A + \tan A\end{aligned}$$

Hence Proved.

38. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A}} = 2 \operatorname{cosec} A$$

Answer

$$\begin{aligned}\sqrt{\frac{1 - \cos A}{1 + \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A}} &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} + \sqrt{\frac{(1 + \cos A)^2}{1 - \cos^2 A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1 + \cos A)^2}{\sin^2 A}} \\ &= \frac{(1 - \cos A)}{\sin A} + \frac{(1 + \cos A)}{\sin A} \\ &= \frac{1}{\sin A} - \frac{\cos A}{\sin A} + \frac{1}{\sin A} + \frac{\cos A}{\sin A} \\ &= \frac{2}{\sin A} \\ &= 2 \operatorname{cosec} A\end{aligned}$$

Hence Proved.

39. Question

Prove the following trigonometric identities:

$$(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

Answer

$$\begin{aligned}
(\sec A - \tan A)^2 &= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right)^2 \\
&= \frac{(1 - \sin A)^2}{\cos^2 A} \\
&= \frac{(1 - \sin A)^2}{1 - \sin^2 A} \\
&= \frac{(1 - \sin A)^2}{(1 - \sin A)(1 + \sin A)} \\
&= \frac{1 - \sin A}{1 + \sin A}
\end{aligned}$$

Hence Proved.

40. Question

Prove the following trigonometric identities:

$$\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$$

Answer

Given: $\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$

To prove: Above equality

Proof: Rationalize the LHS, Use $\sin^2 x + \cos^2 x = 1$ Solve,

$$\begin{aligned}
\frac{1 - \cos A}{1 + \cos A} &= \frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\
&= \frac{(1 - \cos A)^2}{1 - \cos^2 A} \\
&= \frac{(1 - \cos A)^2}{\sin^2 A} \\
&= \left(\frac{1 - \cos A}{\sin A} \right)^2 \\
&= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2 \\
&= (\cot A - \operatorname{cosec} A)^2
\end{aligned}$$

Hence proved

41. Question

Prove the following trigonometric identities:

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$$

Answer

$$\begin{aligned}
\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} &= \frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1} \\
&= \frac{2 \sec A}{\tan^2 A} \\
&= \frac{2}{\cos A} \times \frac{\cos A}{\sin^2 A} \\
&= 2 \operatorname{cosec} A \cot A
\end{aligned}$$

Hence Proved.

42. Question

Prove the following trigonometric identities:

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

Answer

$$\begin{aligned}
 \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} &= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} \\
 &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \\
 &= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\
 &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \\
 &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\
 &= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A} \\
 &= \sin A + \cos A
 \end{aligned}$$

Hence Proved.

43. Question

Prove the following trigonometric identities:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$$

Answer

$$\begin{aligned}
 \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} &= \frac{\operatorname{cosec} A (\operatorname{cosec} A + 1) + \operatorname{cosec} A (\operatorname{cosec} A - 1)}{\operatorname{cosec}^2 A - 1} \\
 &= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1} \\
 &= \frac{2 \operatorname{cosec}^2 A}{\cot^2 A} = \frac{2}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A} = 2 \sec^2 A
 \end{aligned}$$

Hence Proved.

44. Question

Prove the following trigonometric identities:

$$(1 + \tan^2 A) + \left(1 + \frac{1}{\tan^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$$

Answer

$$\begin{aligned}
 (1 + \tan^2 A) + \left(1 + \frac{1}{\tan^2 A}\right) &= \sec^2 A + \frac{1 + \tan^2 A}{\tan^2 A} \\
 &= \sec^2 A + \frac{\sec^2 A}{\tan^2 A} \\
 &= \frac{1}{\cos^2 A} + \frac{1}{\cos^2 A} \times \frac{\cos^2 A}{\sin^2 A} \\
 &= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \\
 &= \frac{1}{1 - \sin^2 A} + \frac{1}{\sin^2 A} \\
 &= \frac{\sin^2 A + 1 - \sin^2 A}{(1 - \sin^2 A) \sin^2 A} \\
 &= \frac{1}{\sin^2 A - \sin^4 A}
 \end{aligned}$$

Hence Proved.

45. Question

Prove the following trigonometric identities:

$$\frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} = 1$$

Answer

$$\begin{aligned}
 \frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} &= \frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\operatorname{cosec}^2 A} \\
 &= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} \\
 &= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \\
 &= \sin^2 A + \cos^2 A \\
 &= 1
 \end{aligned}$$

Hence Proved.

46. Question

Prove the following trigonometric identities:

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

Answer

$$\begin{aligned}
 \frac{\cot A - \cos A}{\cot A + \cos A} &= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \\
 &= \frac{\cos A \left(\frac{1}{\sin A} - 1 \right)}{\cos A \left(\frac{1}{\sin A} + 1 \right)} \\
 &= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}
 \end{aligned}$$

Hence Proved.

47 A. Question

Prove the following trigonometric identities:

$$\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$$

Answer

$$\begin{aligned}
 \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} &= \frac{\{(1 + \cos \theta) + \sin \theta\}}{\{(1 + \cos \theta) - \sin \theta\}} \times \frac{\{(1 + \cos \theta) + \sin \theta\}}{\{(1 + \cos \theta) + \sin \theta\}} \\
 &= \frac{\{(1 + \cos \theta) + \sin \theta\}^2}{(1 + \cos \theta)^2 - \sin^2 \theta} \\
 &= \frac{(1 + \cos \theta)^2 + \sin^2 \theta + 2(1 + \cos \theta) \sin \theta}{(1 + \cos \theta)^2 - \sin^2 \theta} \\
 &= \frac{1 + \cos^2 \theta + 2 \cos \theta + \sin^2 \theta + 2 \sin \theta + 2 \sin \theta \cos \theta}{1 + \cos^2 \theta + 2 \cos \theta - \sin^2 \theta} \\
 &= \frac{1 + \cos^2 \theta + \sin^2 \theta + 2 \cos \theta + 2 \sin \theta + 2 \sin \theta \cos \theta}{1 + \cos^2 \theta + 2 \cos \theta - 1 + \cos^2 \theta} \\
 &= \frac{1 + 1 + 2 \sin \theta + 2 \cos \theta + 2 \sin \theta \cos \theta}{2 \cos^2 \theta + 2 \cos \theta} \\
 &= \frac{2(1 + \sin \theta) + 2 \cos \theta(1 + \sin \theta)}{2 \cos \theta(1 + \cos \theta)} \\
 &= \frac{2(1 + \sin \theta)(1 + \cos \theta)}{2 \cos \theta(1 + \cos \theta)} \\
 &= \frac{1 + \sin \theta}{\cos \theta}
 \end{aligned}$$

Hence Proved.

47 B. Question

Prove the following trigonometric identities:

$$\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$$

Answer

To Prove: $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$

$$\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing the numerator and denominator by $\cos\theta$, we get,

$$\begin{aligned} &= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}} \\ &= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} \\ &= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} \\ &= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \end{aligned}$$

Now, we know that, $\sec^2\theta - \tan^2\theta = 1$

Therefore, replacing 1 by $\sec^2\theta - \tan^2\theta$ in the numerator only, we get,

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

As we know, $a^2 - b^2 = (a-b)(a+b)$

$$\begin{aligned} &= \frac{(\tan\theta + \sec\theta) - \left[(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1} \\ &= \frac{(\tan\theta + \sec\theta) \left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1} \\ &= \frac{(\tan\theta + \sec\theta) [1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1} \end{aligned}$$

$= \sec\theta + \tan\theta$ Now, multiplying and dividing by $\sec\theta - \tan\theta$, we get,

$$\begin{aligned} &= \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} \times (\sec\theta - \tan\theta) \\ &= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} \end{aligned}$$

As we know, $\sec^2\theta - \tan^2\theta = 1$

$$= \frac{1}{\sec\theta - \tan\theta}$$

= R.H.S Hence, proved.

47 C. Question

Prove the following trigonometric identities:

$$\frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \operatorname{cosec} \theta + \cot \theta$$

Answer

$$\begin{aligned} \text{L.H.S} &= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} \\ &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}} \\ &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\ &= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}} \\ &= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \\ &= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2 \cot A - 2 \operatorname{cosec} A + 2 \cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2 \operatorname{cosec} A)} \\ &= \frac{2 \operatorname{cosec}^2 A + 2 \cot A \operatorname{cosec} A - 2 \cot A - 2 \operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2 \operatorname{cosec} A} \\ &= \frac{2 \operatorname{cosec} A (\operatorname{cosec} A + \cot A) - 2 (\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2 \operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A) (2 \operatorname{cosec} A - 2)}{-1 - 1 + 2 \operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A) (2 \operatorname{cosec} A - 2)}{(2 \operatorname{cosec} A - 2)} \\ &= \operatorname{cosec} A + \cot A \end{aligned}$$

Hence Proved.

48. Question

Prove the following trigonometric identities:

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

Answer

To prove: $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Proof: Consider LHS,

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A}$$

Use the formula: $\sec \theta = 1/\cos \theta$ and $\tan \theta = \sin \theta / \cos \theta$

$$\begin{aligned} &= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{\cos A} \\ &= \frac{\cos A}{1 + \sin A} - \frac{1}{\cos A} \end{aligned}$$

Do rationalization,

$$= \left(\frac{\cos A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A} \right) - \frac{1}{\cos A}$$

$$= \frac{\cos A(1 - \sin A)}{(1 + \sin A)(1 - \sin A)} - \frac{1}{\cos A}$$

$$= \frac{\cos A(1 - \sin A)}{(1 - \sin^2 A)} - \frac{1}{\cos A}$$

Use the formula $\cos^2 \theta + \sin^2 \theta = 1$

$$= \frac{\cos A(1 - \sin A)}{\cos^2 A} - \frac{1}{\cos A}$$

$$= \frac{(1 - \sin A)}{\cos A} - \frac{1}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} - \frac{1}{\cos A}$$

= - tan A Consider RHS, $\frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Use the formula: $\sec \theta = 1/\cos \theta$ and $\tan \theta = \sin \theta / \cos \theta$

$$= \frac{1}{\cos A} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\cos A}{1 - \sin A}$$

Do rationalization,

$$= \frac{1}{\cos A} - \left(\frac{\cos A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A} \right)$$

$$= \frac{1}{\cos A} - \frac{\cos A(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}$$

$$= \frac{1}{\cos A} - \frac{\cos A(1 + \sin A)}{(1 - \sin^2 A)}$$

Use the formula $\cos^2 \theta + \sin^2 \theta = 1$

$$= \frac{1}{\cos A} - \frac{\cos A(1 + \sin A)}{\cos^2 A}$$

$$= \frac{1}{\cos A} - \frac{(1 + \sin A)}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

= - tan A LHS = RHS Hence proved.

49. Question

Prove the following trigonometric identities:

$$\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$$

Answer

$$\begin{aligned} RHS &= \sec^2 A \operatorname{cosec}^2 A - 2 \\ &= (1 + \tan^2 A)(1 + \cot^2 A) - 2 \\ &= (\tan^2 A + \cot^2 A + \tan^2 A \cot^2 A + 1) - 2 \\ &= (\tan^2 A + \cot^2 A + 1 + 1) - 2 \\ &= \tan^2 A + \cot^2 A \\ &= LHS \end{aligned}$$

Hence Proved.

50. Question

Prove the following trigonometric identities:

$$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$$

Answer

To prove:

$$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$$

Use the formula

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned} \frac{1 - \tan^2 A}{\cot^2 A - 1} &= \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1} = \\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} \\ &= \tan^2 A \end{aligned}$$

Hence Proved.

51. Question

Prove the following trigonometric identities:

$$1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} = \operatorname{cosec} \theta$$

Answer

$$\begin{aligned} 1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} &= 1 + \frac{\operatorname{cosec}^2 \theta - 1}{1 + \operatorname{cosec} \theta} \\ &= 1 + \frac{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}{(1 + \operatorname{cosec} \theta)} \\ &= 1 + \operatorname{cosec} \theta - 1 \\ &= \operatorname{cosec} \theta \end{aligned}$$

Hence Proved.

52. Question

Prove the following trigonometric identities:

$$\frac{\cos \theta}{\operatorname{cosec} \theta + 1} + \frac{\cos \theta}{\operatorname{cosec} \theta - 1} = 2 \tan \theta$$

Answer

$$\begin{aligned}\frac{\cos \theta}{\operatorname{cosec} \theta + 1} + \frac{\cos \theta}{\operatorname{cosec} \theta - 1} &= \frac{\cos \theta (\operatorname{cosec} \theta - 1) + \cos \theta (\operatorname{cosec} \theta + 1)}{(\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)} \\ &= \frac{2 \cot \theta}{\cot^2 \theta} \\ &= 2 \tan \theta\end{aligned}$$

Hence Proved.

53. Question

Prove the following trigonometric identities:

$$\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} = \cot \theta$$

Answer

$$\begin{aligned}\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} &= \frac{(1 + \cos \theta) - (1 - \cos^2 \theta)}{\sin \theta (1 + \cos \theta)} \\ &= \frac{(1 + \cos \theta) - (1 + \cos \theta)(1 - \cos \theta)}{\sin \theta (1 + \cos \theta)} \\ &= \frac{(1 + \cos \theta) \{1 - (1 - \cos \theta)\}}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\cos \theta}{\sin \theta} \\ &= \cot \theta\end{aligned}$$

Hence Proved.

54. Question

Prove the following trigonometric identities:

$$\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$$

Answer

$$\begin{aligned}\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} &= \frac{\tan^3 \theta}{\sec^2 \theta} + \frac{\cot^3 \theta}{\operatorname{cosec}^2 \theta} \\ &= \tan^3 \theta \cos^2 \theta + \cot^3 \theta \sin^2 \theta \\ &= \frac{\sin^3 \theta}{\cos^3 \theta} \cos^2 \theta + \frac{\cos^3 \theta}{\sin^3 \theta} \sin^2 \theta \\ &= \frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta} \\ &= \frac{\sin^4 \theta + \cos^4 \theta}{\sin \theta \cos \theta} \\ &= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} - 2 \sin \theta \cos \theta \\ &= \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta\end{aligned}$$

Hence Proved.

55. Question

If $T_n = \sin^n \theta + \cos^n \theta$, prove that $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$

Answer

$$T_n = \sin^n \theta + \cos^n \theta$$

$$T_1 = \sin^1 \theta + \cos^1 \theta = \sin \theta + \cos \theta$$

$$T_3 = \sin^3 \theta + \cos^3 \theta, T_5 = \sin^5 \theta + \cos^5 \theta \text{ and } T_7 = \sin^7 \theta + \cos^7 \theta$$

Now, we have,

$$\begin{aligned} \frac{T_3 - T_5}{T_1} &= \frac{\sin^3 \theta + \cos^3 \theta - (\sin^5 \theta + \cos^5 \theta)}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta + \cos^3 \theta - \sin^5 \theta - \cos^5 \theta}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta (1 - \sin^2 \theta) + \cos^3 \theta (1 - \cos^2 \theta)}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta \cos^2 \theta + \cos^3 \theta \sin^2 \theta}{\sin \theta + \cos \theta} \\ &= \frac{\sin^2 \theta \cos^2 \theta (\sin \theta + \cos \theta)}{\sin \theta + \cos \theta} \\ &= \sin^2 \theta \cos^2 \theta \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \frac{T_5 - T_7}{T_3} &= \frac{\sin^5 \theta + \cos^5 \theta - (\sin^7 \theta + \cos^7 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta + \cos^5 \theta - \sin^7 \theta - \cos^7 \theta}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta (1 - \sin^2 \theta) + \cos^5 \theta (1 - \cos^2 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta \cos^2 \theta + \cos^5 \theta \sin^2 \theta}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^2 \theta \cos^2 \theta (\sin^3 \theta + \cos^3 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \sin^2 \theta \cos^2 \theta \quad \dots (2) \end{aligned}$$

From (1) and (2), we get

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

Hence Proved.

56. Question

Prove the following trigonometric identities:

$$\left(\tan \theta + \frac{1}{\cos \theta} \right)^2 + \left(\tan \theta - \frac{1}{\cos \theta} \right)^2 = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right)$$

Answer

$$\begin{aligned} &\left(\tan \theta + \frac{1}{\cos \theta} \right)^2 + \left(\tan \theta - \frac{1}{\cos \theta} \right)^2 \\ &= (\tan \theta + \sec \theta)^2 + (\tan \theta - \sec \theta)^2 \\ &= (\tan^2 \theta + \sec^2 \theta + 2 \tan \theta \sec \theta) + (\tan^2 \theta + \sec^2 \theta - 2 \tan \theta \sec \theta) \\ &= \tan^2 \theta + \sec^2 \theta + 2 \tan \theta \sec \theta + \tan^2 \theta + \sec^2 \theta - 2 \tan \theta \sec \theta \\ &= 2(\tan^2 \theta + \sec^2 \theta) \\ &= 2 \left(\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{1}{\cos^2 \theta} \right) \\ &= 2 \left(\frac{1 + \sin^2 \theta}{\cos^2 \theta} \right) = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right) \end{aligned}$$

Hence Proved.

57. Question

Prove the following trigonometric identities:

$$\left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}$$

Answer

$$\begin{aligned}
& \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\frac{1}{\sin^2 \theta} - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{1 - \cos^4 \theta} + \frac{\sin^2 \theta}{1 - \sin^4 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{\cos^2 \theta + \sin^2 \theta - \cos^4 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta + \sin^2 \theta - \sin^4 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{\cos^2 \theta (1 - \cos^2 \theta) + \sin^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta + \sin^2 \theta (1 - \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{\cos^2 \theta \sin^2 \theta + \sin^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta + \sin^2 \theta \cos^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{\sin^2 \theta (\cos^2 \theta + 1)} + \frac{\sin^2 \theta}{\cos^2 \theta (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (\cos^2 \theta + 1)}{\sin^2 \theta \cos^2 \theta (\cos^2 \theta + 1) (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (\cos^2 \theta + 1)}{(\cos^2 \theta + 1) (1 + \sin^2 \theta)} \\
&= \frac{(\cos^4 \theta + \sin^2 \theta \cos^4 \theta) + (\sin^4 \theta \cos^2 \theta + \sin^6 \theta)}{1 + \cos^2 \theta + \sin^2 \theta + \cos^2 \theta \sin^2 \theta} \\
&= \frac{(\cos^4 \theta + \sin^4 \theta) + \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{1 + 1 + \cos^2 \theta \sin^2 \theta} \\
&= \frac{1 - 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta \times 1}{2 + \cos^2 \theta \sin^2 \theta} \\
&= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}
\end{aligned}$$

Hence Proved.

58. Question

Prove the following trigonometric identities:

$$\left(\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} \right)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Answer

$$\begin{aligned}
\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 &= \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} \times \frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2 \\
&= \left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}\right]^2 \\
&= \left[\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}\right]^2 \\
&= \left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2 \\
&= \left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2 \\
&= \left[\frac{2(1+\sin\theta)-2\cos\theta(1+\sin\theta)}{2\sin^2\theta+2\sin\theta}\right]^2 \\
&= \left[\frac{2(1+\sin\theta)(1-\cos\theta)}{2\sin\theta(1+\sin\theta)}\right]^2 \\
&= \frac{(1-\cos\theta)^2}{\sin^2\theta} \\
&= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \\
&= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \\
&= \frac{1-\cos\theta}{1+\cos\theta}
\end{aligned}$$

Hence Proved.

59. Question

Prove the following trigonometric identities:

$$(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2 \tan A$$

Answer

$$\begin{aligned}
&(\sec A + \tan A - 1)(\sec A - \tan A + 1) \\
&= [\sec A + \tan A - (\sec^2 A - \tan^2 A)][\sec A - \tan A + (\sec^2 A - \tan^2 A)] \\
&= [\sec A + \tan A - (\sec A - \tan A)(\sec A + \tan A)][\sec A - \tan A + (\sec A - \tan A)(\sec A + \tan A)] \\
&= (\sec A + \tan A)[1 - (\sec A - \tan A)](\sec A - \tan A)[1 + (\sec A + \tan A)] \\
&= (\sec A + \tan A)(\sec A - \tan A)[1 - \sec A + \tan A][1 + \sec A + \tan A] \\
&= (\sec^2 A - \tan^2 A)[1 - \sec A + \tan A][1 + \sec A + \tan A] \\
&= 1 \times [1 - \sec A + \tan A][1 + \sec A + \tan A] \\
&= \left[1 - \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right]\left[1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right] \\
&= \left[\frac{\cos A + \sin A - 1}{\cos A}\right]\left[\frac{\cos A + \sin A + 1}{\cos A}\right] \\
&= \left[\frac{(\cos A + \sin A)^2 - 1}{\cos^2 A}\right] \\
&= \left[\frac{\cos^2 A + \sin^2 A + 2\sin A \cos A - 1}{\cos^2 A}\right] \\
&= \left[\frac{1 + 2\sin A \cos A - 1}{\cos^2 A}\right] \\
&= \left[\frac{2\sin A}{\cos A}\right] \\
&= 2 \tan A
\end{aligned}$$

Hence Proved.

60. Question

Prove the following trigonometric identities:

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$$

Answer

$$\begin{aligned}
&= \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \\
&= \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \\
&= \frac{[(\sin \theta + \cos \theta) - 1][(\sin \theta + \cos \theta) + 1]}{\sin \theta \cdot \cos \theta} \\
&= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cdot \cos \theta} \\
&= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cdot \cos \theta} \\
&= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cdot \cos \theta} \\
&= \frac{2 \sin \theta \cos \theta}{\sin \theta \cdot \cos \theta} = 2 = \text{R.H.S.}
\end{aligned}$$

Hence Proved.

61. Question

Prove the following trigonometric identities:

$$(\operatorname{cosec} \theta - \sec \theta)(\cot \theta - \tan \theta) = (\operatorname{cosec} \theta + \sec \theta)(\sec \theta \operatorname{cosec} \theta - 2)$$

Answer

$$\begin{aligned}
LHS &= (\operatorname{cosec} \theta - \sec \theta)(\cot \theta - \tan \theta) \\
&= \left(\frac{1}{\sin \theta} - \frac{1}{\cos \theta}\right) \left(\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}\right) \\
&= \left(\frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta}\right) \left(\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}\right) \\
&= \left(\frac{(\cos \theta - \sin \theta)^2 (\cos \theta + \sin \theta)}{\sin^2 \theta \cos^2 \theta}\right)
\end{aligned}$$

$$\begin{aligned}
RHS &= (\operatorname{cosec} \theta + \sec \theta)(\sec \theta \operatorname{cosec} \theta - 2) \\
&= \left(\frac{1}{\sin \theta} + \frac{1}{\cos \theta}\right) \left(\frac{1}{\sin \theta} \times \frac{1}{\cos \theta} - 2\right) \\
&= \left(\frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}\right) \left(\frac{1 - 2 \sin \theta \cos \theta}{\sin \theta \cos \theta}\right) \\
&= \left(\frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}\right) \left(\frac{\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta}{\sin \theta \cos \theta}\right) \\
&= \left(\frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}\right) \left(\frac{(\cos \theta - \sin \theta)^2}{\sin \theta \cos \theta}\right) \\
&= \left(\frac{(\cos \theta - \sin \theta)^2 (\cos \theta + \sin \theta)}{\sin^2 \theta \cos^2 \theta}\right)
\end{aligned}$$

Therefore, LHS = RHS

Hence Proved.

62. Question

Prove the following trigonometric identities:

$$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$$

Answer

To prove: $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

Proof: Consider LHS, $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$ We know, $\operatorname{cosec} A = 1/\sin A$, $\sec A = 1/\cos A$, $\tan A = \sin A/\cos A$, $\cot A = \cos A/\sin A$ So,

$$\begin{aligned}
(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) &= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) \\
&= \left(\frac{\sin A - \cos A}{\cos A \sin A}\right) \left(\frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\cos A \sin A}\right)
\end{aligned}$$

Using the formula $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$ we get,

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

$$\begin{aligned} RHS &= \tan A \sec A - \cot A \operatorname{cosec} A \\ &= \frac{\sin A}{\cos A} \times \frac{1}{\cos A} - \frac{\cos A}{\sin A} \times \frac{1}{\sin A} \\ &= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} \\ &= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A} \\ &= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A} \end{aligned}$$

LHS = RHS

Hence Proved.

63. Question

Prove the following trigonometric identities:

$$\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$$

Answer

$$\begin{aligned} &\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} \\ &= \frac{\cos A \times \frac{1}{\sin A} - \sin A \times \frac{1}{\cos A}}{\cos A + \sin A} \\ &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A} \\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A} \\ &= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\sin A \cos A (\cos A + \sin A)} \\ &= \frac{(\cos A - \sin A)}{\sin A \cos A} \\ &= \frac{1}{\sin A} - \frac{1}{\cos A} \\ &= \operatorname{cosec} A - \sec A \end{aligned}$$

Hence Proved.

64. Question

Prove the following trigonometric identities:

$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\operatorname{cosec} A + \cot A - 1} = 1$$

Answer

$$\begin{aligned}
& \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\operatorname{cosec} A + \cot A - 1} \\
&= \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1} \\
&= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A} \\
&= \sin A \cos A \left(\frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A - \sin A} \right) \\
&= \sin A \cos A \left(\frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{(1 + \sin A - \cos A)(1 + \cos A - \sin A)} \right) \\
&= \sin A \cos A \left(\frac{2}{1 - \sin^2 A - \cos^2 A + 2 \sin A \cos A} \right) \\
&= \sin A \cos A \left(\frac{2}{1 - (\sin^2 A + \cos^2 A) + 2 \sin A \cos A} \right) \\
&= \sin A \cos A \left(\frac{2}{1 - 1 + 2 \sin A \cos A} \right) \\
&= \sin A \cos A \left(\frac{1}{\sin A \cos A} \right) \\
&= 1
\end{aligned}$$

Hence Proved.

65. Question

Prove the following trigonometric identities:

$$\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \cos A$$

Answer

$$\begin{aligned}
\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} &= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2} \\
&= \frac{\sin A}{\cos A} \times \cos^4 A + \frac{\cos A}{\sin A} \times \sin^4 A \\
&= \sin A \cos^3 A + \cos A \sin^3 A \\
&= \sin A \cos A (\cos^2 A + \sin^2 A) \\
&= \sin A \cos A \times 1 \\
&= \sin A \cos A
\end{aligned}$$

Hence Proved.

66. Question

Prove the following trigonometric identities:

$$\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$$

Answer

$$\begin{aligned}
& \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A \\
&= \sec^4 A - \sec^4 A \sin^4 A - 2 \tan^2 A \\
&= \sec^4 A - \frac{\sin^4 A}{\cos^4 A} - 2 \tan^2 A \\
&= (\sec^2 A)^2 - \tan^4 A - 2 \tan^2 A \\
&= (1 + \tan^2 A)^2 - \tan^4 A - 2 \tan^2 A \\
&= 1 + \tan^4 A + 2 \tan^2 A - \tan^4 A - 2 \tan^2 A \\
&= 1
\end{aligned}$$

Hence Proved.

67. Question

Prove the following trigonometric identities:

$$\frac{\cot^2 A (\sec A - 1)}{1 + \sin A} = \sec^2 A \left(\frac{1 - \sin A}{1 + \sec A} \right)$$

Answer

$$\begin{aligned}
\frac{\cot^2 A (\sec A - 1)}{1 + \sin A} &= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1}{\cos A} - 1 \right)}{1 + \sin A} \\
&= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1 - \cos A}{\cos A} \right)}{1 + \sin A} \\
&= \frac{\frac{\cos A}{1 - \cos^2 A} (1 - \cos A)}{1 + \sin A} \\
&= \frac{\cos A}{(1 - \cos A)(1 + \cos A)} (1 - \cos A) \\
&= \frac{\cos A}{(1 + \cos A)(1 + \sin A)} \quad \dots (1)
\end{aligned}$$

$$\begin{aligned}
RHS &= \frac{1}{\cos^2 A} \left(\frac{1 - \sin A}{1 + \frac{1}{\cos A}} \right) \\
&= \frac{1}{\cos^2 A} \frac{(1 - \sin A) \cos A}{(1 + \cos A)} \\
&= \frac{1}{\cos A} \frac{(1 - \sin A)}{(1 + \cos A)} \times \frac{(1 + \sin A)}{(1 + \sin A)} \\
&= \frac{1}{\cos A} \frac{(1 - \sin^2 A)}{(1 + \cos A)(1 + \sin A)} \\
&= \frac{1}{\cos A} \frac{\cos^2 A}{(1 + \cos A)(1 + \sin A)}
\end{aligned}$$

$$= \frac{\cos A}{(1 + \cos A)(1 + \sin A)} \quad \dots (2)$$

From (1) and (2) we get

$$LHS = RHS$$

Hence Proved.

68. Question

Prove the following trigonometric identities:

$$(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

Answer

$$\textbf{To Prove:} (1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

Proof: Consider the LHS,

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \left(\frac{1}{\cos A} \times \sin^2 A \right) - \left(\frac{1}{\sin A} \times \cos^2 A \right)$$

$$= \sin A - \cos A + \cot A \sin A - \cot A \cos A + \tan A \sin A - \tan A \cos A$$

Use the formula:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= \sin A - \cos A + \left(\frac{\cos A}{\sin A} \times \sin A \right) - \left(\frac{\cos A}{\sin A} \times \cos A \right) + \left(\frac{\sin A}{\cos A} \times \sin A \right) - \left(\frac{\sin A}{\cos A} \times \cos A \right)$$

$$= \sin A - \cos A + \cos A - \frac{\cos^2 A}{\sin A} + \frac{\sin^2 A}{\cos A} - \sin A$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

We know:

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} \text{ and } \cos \theta = \frac{1}{\sec \theta}$$

So,

$$(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$$

Again use the formula:

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} \text{ and } \cos \theta = \frac{1}{\sec \theta}$$

So,

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \left(\frac{1}{\cos A} \times \sin^2 A \right) - \left(\frac{1}{\sin A} \times \cos^2 A \right)$$

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \frac{\sin A \times \sin A}{\cos A} - \frac{\cos A \cos A}{\sin A}$$

Use the formula:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

$$\text{Therefore, } (1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

Hence Proved.

69. Question

Prove the following trigonometric identities:

$$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$$

Answer

To prove: $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$ **Proof:** Take LHS, Use the identity $\sin^2 \theta + \cos^2 \theta = 1$

$$\begin{aligned} \sin^2 A \cos^2 B - \cos^2 A \sin^2 B &= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B \\ \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B &= \sin^2 A - \sin^2 A \end{aligned}$$

$$= \sin^2 A - \sin^2 B$$

$$= \text{RHS} \text{ Hence Proved}$$

70. Question

Prove the following trigonometric identities:

$$\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$$

Answer

Use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\begin{aligned}
 \text{L.H.S} &= \frac{\cot A + \tan B}{\cot B + \tan A} \\
 &= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} \\
 &= \frac{\frac{\cos A \cot B \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \sin B}} \\
 &= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\cos A \sin B}{\cos A \cos B + \sin A \sin B} \\
 &= \frac{\cos A \sin B}{\sin A \cos B} \\
 &= \cot A \tan B \\
 &= \text{R.H.S}
 \end{aligned}$$

Hence Proved.

71. Question

Prove the following trigonometric identities:

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

Answer

$$\begin{aligned}
 \frac{\tan A + \tan B}{\cot A + \cot B} &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} \\
 &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin B \cos A + \cos B \sin A}{\sin A \sin B}} \\
 &= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \times \frac{\sin A \sin B}{\sin B \cos A + \cos B \sin A} \\
 &= \frac{\sin A \sin B}{\cos A \cos B} \\
 &= \tan A \tan B
 \end{aligned}$$

Hence Proved.

72. Question

Prove the following trigonometric identities:

$$\cot^2 A \cos ec^2 B - \cot^2 B \cos ec^2 A = \cot^2 A - \cot^2 B$$

Answer

$$\begin{aligned}
 &\cot^2 A \cos ec^2 B - \cot^2 B \cos ec^2 A \\
 &= \cot^2 A (1 + \cot^2 B) - \cot^2 B (1 + \cot^2 A) \\
 &= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 A \cot^2 B \\
 &= \cot^2 A - \cot^2 B
 \end{aligned}$$

Hence Proved.

73. Question

Prove the following trigonometric identities:

$$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$$

Answer

$$\begin{aligned}
& \tan^2 A \sec^2 B - \sec^2 A \tan^2 B \\
&= \tan^2 A (1 + \tan^2 B) - (1 + \tan^2 A) \tan^2 B \\
&= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 A \tan^2 B \\
&= \tan^2 A - \tan^2 B
\end{aligned}$$

Hence Proved.

74. Question

If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$, prove that $x^2 - y^2 = a^2 - b^2$

$$x^2 - y^2 = a^2 - b^2$$

Answer

$$\begin{aligned}
x &= a \sec \theta + b \tan \theta \\
\Rightarrow x^2 &= (a \sec \theta + b \tan \theta)^2 = a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta \\
y &= a \tan \theta + b \sec \theta \\
\Rightarrow y^2 &= (a \tan \theta + b \sec \theta)^2 = a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \tan \theta \sec \theta \\
\text{Now,} \\
x^2 - y^2 &= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \tan \theta \sec \theta \\
x^2 - y^2 &= a^2 \sec^2 \theta + b^2 \tan^2 \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta \\
x^2 - y^2 &= a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta) \\
x^2 - y^2 &= (a^2 - b^2) (\sec^2 \theta - \tan^2 \theta) \\
x^2 - y^2 &= (a^2 - b^2) \times 1 \\
x^2 - y^2 &= a^2 - b^2
\end{aligned}$$

Hence Proved.

75. Question

$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ and $\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

Answer

$$\begin{aligned}
\left(\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta \right)^2 + \left(\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta \right)^2 &= 2 \\
\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + 2 \frac{xy}{ab} \cos \theta \sin \theta + \frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta - 2 \frac{xy}{ab} \sin \theta \cos \theta &= 2 \\
\frac{x^2}{a^2} \cos^2 \theta + \frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta &= 2 \\
\frac{x^2}{a^2} (\cos^2 \theta + \sin^2 \theta) + \frac{y^2}{b^2} (\sin^2 \theta + \cos^2 \theta) &= 2 \\
\frac{x^2}{a^2} \times 1 + \frac{y^2}{b^2} \times 1 &= 2 \\
\frac{x^2}{a^2} + \frac{y^2}{b^2} &= 2
\end{aligned}$$

Hence Proved.

76. Question

If $\operatorname{cosec} \theta - \sin \theta = a^3$, $\sec \theta - \cos \theta = b^3$, prove that $a^2 b^2 (a^2 + b^2)$.

Answer

$$\begin{aligned}
\frac{1}{\sin \theta} - \sin \theta &= a^3 \\
\frac{1 - \sin^2 \theta}{\sin \theta} &= a^3 \\
\frac{\cos^2 \theta}{\sin \theta} &= a^3 \\
a^3 &= \frac{\cos^2 \theta}{\sin \theta} \\
a &= \frac{\cos^{2/3} \theta}{\sin^{1/3} \theta} \\
\Rightarrow a^2 &= \frac{\cos^{4/3} \theta}{\sin^{2/3} \theta} \quad \dots (1)
\end{aligned}$$

Similarly we can see that,

$$\sec\theta - \cos\theta = b^3$$

$$\frac{1}{\cos\theta} - \cos\theta = b^3$$

$$\frac{1 - \cos^2\theta}{\cos\theta} = b^3$$

$$b^3 = \frac{\sin^2\theta}{\cos\theta}$$

$$b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

$$b^2 = \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta} \dots (2)$$

From (1) and (2), we get

$$\begin{aligned} a^2 b^2 (a^2 + b^2) &= \cos^{\frac{4}{3}-\frac{2}{3}}\theta \sin^{\frac{4}{3}-\frac{2}{3}}\theta \left(\frac{\cos^2\theta + \sin^2\theta}{\sin^{2/3}\theta \cos^{2/3}\theta} \right) \\ &= \cos^{\frac{2}{3}}\theta \sin^{\frac{2}{3}}\theta \left(\frac{1}{\sin^{2/3}\theta \cos^{2/3}\theta} \right) \\ &= 1 \end{aligned}$$

Hence Proved.

77. Question

If $a \cos^3 \theta + 3a \cos \theta \sin^2 \theta = m$, $a \sin^3 \theta + 3a \cos^2 \theta \sin \theta = n$, prove that

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}.$$

Answer

$$\begin{aligned} (m+n)^{2/3} + (m-n)^{2/3} &= 2a^{2/3} \\ &= \left(a \cos^3 \theta + 3a \cos \theta \sin^2 \theta + a \sin^3 \theta + 3a \cos^2 \theta \sin \theta \right)^{2/3} \\ &\quad + \left(a \cos^3 \theta + 3a \cos \theta \sin^2 \theta + a \sin^3 \theta + 3a \cos^2 \theta \sin \theta \right)^{2/3} \\ &= a^{2/3} \left(\cos^3 \theta + 3 \cos \theta \sin^2 \theta + \sin^3 \theta + 3 \cos^2 \theta \sin \theta \right)^{2/3} \\ &\quad + a^{2/3} \left(a \cos^3 \theta + 3a \cos \theta \sin^2 \theta + a \sin^3 \theta + 3a \cos^2 \theta \sin \theta \right)^{2/3} \\ &= a^{2/3} \left[(\cos \theta + \sin \theta)^3 \right]^{2/3} + a^{2/3} \left[(\cos \theta - \sin \theta)^3 \right]^{2/3} \\ &= a^{2/3} (\cos \theta + \sin \theta)^2 + a^{2/3} (\cos \theta - \sin \theta)^2 \\ &= a^{2/3} (\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta) + a^{2/3} (\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta) \\ &= a^{2/3} [(1 + 2 \sin \theta \cos \theta) + (1 - 2 \sin \theta \cos \theta)] \\ &= a^{2/3} \times 2 \\ &= 2a^{2/3} \end{aligned}$$

Hence Proved.

78. Question

If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

Answer

$$\begin{aligned} x = a \cos^3 \theta &\Rightarrow \frac{x}{a} = \cos^3 \theta \\ y = b \sin^3 \theta &\Rightarrow \frac{y}{b} = \sin^3 \theta \\ \text{Now, } \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} &= (\cos^3 \theta)^{2/3} + (\sin^3 \theta)^{2/3} \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \\ \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} &= 1 \end{aligned}$$

79. Question

If $3 \sin \theta + 5 \cos \theta = 5$, prove that $5 \sin \theta - 3 \cos \theta = \pm 3$.

Answer

$$\begin{aligned}
& 3\sin\theta + 5\cos\theta = 5 \\
\Rightarrow & 3\sin\theta = 5 - 5\cos\theta \\
\Rightarrow & 3\sin\theta = 5(1 - \cos\theta) \\
\Rightarrow & 3\sin\theta = \frac{5(1 - \cos\theta) \times (1 + \cos\theta)}{(1 + \cos\theta)} \\
\Rightarrow & 3\sin\theta = \frac{5(1 - \cos^2\theta)}{(1 + \cos\theta)} \\
\Rightarrow & 3\sin\theta = \frac{5\sin^2\theta}{(1 + \cos\theta)} \\
\Rightarrow & 3 = \frac{5\sin\theta}{(1 + \cos\theta)} \\
\Rightarrow & 3 + 3\cos\theta = 5\sin\theta \\
\Rightarrow & 3 = 5\sin\theta - 3\cos\theta
\end{aligned}$$

Hence Proved.

80. Question

If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

Answer

$$\begin{aligned}
m^2 + n^2 &= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 \\
&= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \cos \theta \sin \theta \\
&= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) \\
&= a^2 \times 1 + b^2 \times 1 \\
&= a^2 + b^2
\end{aligned}$$

Hence Proved.

81. Question

If $\operatorname{cosec} \theta + \cot \theta = m$ and $\operatorname{cosec} \theta - \cot \theta = n$, prove that $mn = 1$.

Answer

$$\begin{aligned}
\operatorname{cosec} \theta + \cot \theta &= m \\
\operatorname{cosec} \theta - \cot \theta &= n \\
mn &= (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta) \\
mn &= (\operatorname{cosec}^2 \theta - \cot^2 \theta) \\
mn &= 1
\end{aligned}$$

Hence Proved.

82. Question

If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$

Answer

$$\begin{aligned}
\text{Consider, } \cos A + \cos^2 A = 1 &\Rightarrow \cos A = 1 - \cos^2 A \text{ As we know } 1 - \cos^2 A = \sin^2 A \Rightarrow \cos A = \sin^2 A \dots (1) \\
\text{Now } \sin^2 A + \sin^4 A &= \sin^2 A + (\sin^2 A)^2 \\
&= \sin^2 A + (\cos A)^2 \\
&= \sin^2 A + \cos^2 A \\
&= 1
\end{aligned}$$

Hence Proved.

83. Question

Prove that:

$$\begin{aligned}
\text{(i)} \quad & \sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta \\
\text{(ii)} \quad & \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = 2 \sec \theta \\
\text{(iii)} \quad & \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = 2 \operatorname{cosec} \theta \\
\text{(iv)} \quad & \frac{\sec \theta - 1}{\sec \theta + 1} = \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2
\end{aligned}$$

Answer

(i)

$$\begin{aligned}\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} &= \sqrt{\frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1}} + \sqrt{\frac{\frac{1}{\cos \theta} + 1}{\frac{1}{\cos \theta} - 1}} \\&= \sqrt{\frac{\frac{1 - \cos \theta}{\cos \theta}}{\frac{1 + \cos \theta}{\cos \theta}}} + \sqrt{\frac{\frac{1 + \cos \theta}{\cos \theta}}{\frac{1 - \cos \theta}{\cos \theta}}} \\&= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} + \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} \\&= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta}} \\&= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} + \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} \\&= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} + \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} \\&= \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}} + \sqrt{\frac{(1 + \cos \theta)^2}{\sin^2 \theta}} \\&= \frac{1 - \cos \theta}{\sin \theta} + \frac{1 + \cos \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} + \frac{1}{\sin \theta} \\&= \frac{2}{\sin \theta} \\&= 2 \operatorname{cosec} \theta\end{aligned}$$

Hence Proved.

(ii)

$$\begin{aligned}\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} &= \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}} \\&= \sqrt{\frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta}} + \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} \\&= \sqrt{\frac{(1 + \sin \theta)^2}{\cos^2 \theta}} + \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} \\&= \frac{1 + \sin \theta}{\cos \theta} + \frac{1 - \sin \theta}{\cos \theta} \\&= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \\&= \frac{2}{\cos \theta} \\&= 2 \sec \theta\end{aligned}$$

Hence Proved.

(iii)

$$\begin{aligned}\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} &= \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}} \\&= \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} + \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} \\&= \sqrt{\frac{(1 + \cos \theta)^2}{\sin^2 \theta}} + \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}} \\&= \frac{1 + \cos \theta}{\sin \theta} + \frac{1 - \cos \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \\&= \frac{2}{\sin \theta} \\&= 2 \operatorname{cosec} \theta\end{aligned}$$

Hence Proved.

(iv)

$$\begin{aligned}
 \frac{\sec \theta - 1}{\sec \theta + 1} &= \frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1} \\
 &= \frac{\frac{1 - \cos \theta}{\cos \theta}}{\frac{1 + \cos \theta}{\cos \theta}} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\
 &= \frac{1 - \cos^2 \theta}{(1 + \cos \theta)^2} \\
 &= \frac{\sin^2 \theta}{(1 + \cos \theta)^2} \\
 &= \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2
 \end{aligned}$$

Hence Proved.

84. Question

If $\cos \theta + \cos^2 \theta = 1$, prove that

Answer

$$\sin^{12} \theta + 3 \sin^{10} \theta + 3 \sin^8 \theta + \sin^6 \theta + 2 \sin^4 \theta + 2 \sin^2 \theta - 2 = 1$$

$$\cos \theta + \cos^2 \theta = 1$$

$$\cos = 1 - \cos^2 \theta$$

$$\cos = \sin^2 \theta \quad \text{--- (i)}$$

$$\text{Now, } \sin^{12} \theta + 3 \sin^{10} \theta + 3 \sin^8 \theta + \sin^6 \theta + 2 \sin^4 \theta + 2 \sin^2 \theta - 2$$

$$\begin{aligned}
 &= (\sin^4 \theta)^3 + \sin^4 \theta - \sin^2 \theta [\sin^4 \theta + \sin^2 \theta] \\
 &\quad + (\sin^2 \theta)^3 + 2 (\sin^2 \theta)^2 + 2 \sin^2 \theta - 2
 \end{aligned}$$

$$\text{Using } (a+b)^3 = a^3 + b^3 + 3(a+b)ab \text{ and}$$

$$\text{Also from (i) } \sin^2 \theta \cos^2 \theta$$

$$(\sin^4 \theta + \sin^2 \theta)^3 + 2 (\cos \theta)^2 + 2 \cos \theta - 2$$

$$((\sin^2 \theta)^2 + \sin^2 \theta) + 2 \cos^2 \theta + 2 \cos \theta - 2$$

$$(\cos^2 + \sin^2 \theta)^3 + 2 \cos^2 \theta + 2 \cos \theta - 2$$

$$(\cos)^3 + 2 \cos^2 \theta + 2 \sin^2 \theta - 2$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$1 + 2(\sin^2 \theta + \cos^2 \theta) - 2$$

$$1 + 2(1) - 2 = 1$$

Hence Proved.

85. Question

Given that:

$$(1 + \cos \alpha)(1 + \cos \beta)(1 + \cos \gamma) = (1 - \cos \alpha)(1 - \cos \beta)(1 - \cos \gamma)$$

Show that one of the values of each member of this equality is $\sin \alpha \sin \beta \sin \gamma$

Answer

L.H.S

$$\begin{aligned}\text{we know that } 1 + \cos \theta &= 1 + \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \\ &= 2\cos^2 \frac{\theta}{2}.\end{aligned}$$

$$\therefore \Rightarrow 2\cos^2 \frac{\alpha}{2} - 2\cos^2 \frac{\beta}{2} \cdot 2\cos^2 \frac{\gamma}{2} \dots (i)$$

Multiply (i) with $\sin \alpha \sin \beta \sin \gamma$ and divide with same

$$\text{we get } \frac{8\cos^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2} \cos^2 \frac{\gamma}{2}}{\sin \alpha \sin \beta \sin \gamma} \times \sin \alpha \sin \beta \sin \gamma$$

$$\Rightarrow \frac{8\cos^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2} \cos^2 \frac{\gamma}{2} \times \sin \alpha \sin \beta \sin \gamma}{2 \cdot 2\cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2} \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\gamma}{2}}$$

$$\Rightarrow \sin \alpha \sin \beta \sin \gamma \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}$$

$$\text{R.H.S } (1 - \cos \alpha)(1 - \cos \beta)(1 \cos \gamma)$$

$$\text{we know that } 1 - \cos \theta = 1 - \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 2\sin^2 \frac{\theta}{2}$$

$$\Rightarrow 2\sin^2 \frac{\alpha}{2}, 2\sin^2 \frac{\beta}{2}, 2\sin^2 \frac{\gamma}{2}$$

Multiply and divide by $\sin \alpha \sin \beta \sin \gamma$ we get

$$\frac{2\sin^2 \frac{\alpha}{2} \cdot 2\sin^2 \frac{\beta}{2} \cdot 2\sin^2 \frac{\gamma}{2} \cdot \sin \alpha \sin \beta \sin \gamma}{\sin \alpha \sin \beta \sin \gamma}$$

$$\Rightarrow \frac{2\sin^2 \frac{\alpha}{2} \cdot 2\sin^2 \frac{\beta}{2} \cdot 2\sin^2 \frac{\gamma}{2} \cdot \sin \alpha \sin \beta \sin \gamma}{2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \cdot 2\sin \frac{\beta}{2} \cos \frac{\beta}{2} \cdot 2\sin \frac{\gamma}{2} \cos \frac{\gamma}{2}}$$

$$\Rightarrow \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2} \sin \alpha \sin \beta \sin \gamma$$

hence $\sin \alpha \sin \beta \sin \gamma$ is the member of equality

Hence Proved.

86. Question

$$\text{If } \sin \theta + \cos \theta = x, \text{ prove that } \sin^6 \theta + \cos^6 \theta = \frac{4 - 3(x^2 - 1)^2}{4}$$

Answer

$$\sin \theta + \cos \theta = x$$

Squaring on both sides

$$(\sin \theta + \cos \theta)^2 = x^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = x^2$$

$$\therefore \sin \theta \cos \theta = \frac{x^2 - 1}{2} \dots (1)$$

$$\text{We know } \sin^2 \theta + \cos^2 \theta = 1$$

cubing on both since

$$(\sin^2 \theta + \cos^2 \theta)^3 = (1)^3$$

$$\sin^6 \theta + \cos^6 \theta + 3\sin \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) = 1$$

$$\Rightarrow \sin^6 \theta + \cos^6 \theta = 1 - 3\sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{3(x^2 - 1)^2}{4} \text{ form - (1)}$$

$$\therefore \sin^6 \theta + \cos^6 \theta = \frac{4 - 3(x^2 - 1)^2}{4}$$

Hence proved

Hence Proved.

87. Question

$$\text{If } x = a \sec \theta \cos \phi, \quad y = b \sec \theta \cos \phi, \text{ and } z = c \tan \theta, \text{ show that } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Answer

$$x = a \sec \theta \cos \phi \Rightarrow x^2 = a^2 \sec^2 \theta \cos^2 \phi$$

$$\therefore \frac{x^2}{a^2} = \sec^2 \theta \cos^2 \phi \quad \dots(1)$$

$$y = b \sec \theta \sin \phi \Rightarrow y^2 = b^2 \sec^2 \theta \sin^2 \phi$$

$$\therefore \frac{y^2}{b^2} = \sec^2 \theta \sin^2 \phi \quad \dots(2)$$

$$z = c \tan \theta \Rightarrow z^2 = c^2 \tan^2 \theta$$

$$\therefore \frac{z^2}{c^2} = \tan^2 \theta \quad \dots(3)$$

$$\begin{aligned} \text{Now, } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} &= \sec^2 \theta \cos^2 \phi + \sec^2 \theta \sin^2 \phi - \tan^2 \theta \\ &= \sec^2 \theta (\cos^2 \phi + \sin^2 \phi) - \tan^2 \theta \\ &= \sec^2 \theta \times 1 - \tan^2 \theta \\ &= \sec^2 \theta - \tan^2 \theta = 1 \end{aligned}$$

$$\text{hence, } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hence Proved.

Exercise 6.2

1. Question

If $\cos \theta = \frac{4}{5}$, find all other trigonometric ratios of angle θ .

Answer

$$\sin \theta = 3/5, \tan \theta = 3/4, \sec \theta = 5/4,$$

$$\cos \theta = \frac{4}{5} \Rightarrow \sin \theta = \frac{3}{5}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{4/5} = \frac{5}{4}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

Hence Proved.

2. Question

If $\sin \theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle θ

Answer

$$\cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{1} = 1$$

3. Question

If $\tan \theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \cot^2 \theta}$

Answer

Given, $\tan\theta = \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{P}{B} = \frac{1}{\sqrt{2}}$$

Let $P = k$ and $B = \sqrt{2}k$

Then,

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{k^2 + 2k^2}$$

$$= \sqrt{3k^2}$$

$$= \sqrt{3}k$$

Hence,

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} &= \frac{\operatorname{cosec}^2\theta \left(1 - \frac{\sec^2\theta}{\operatorname{cosec}^2\theta}\right)}{\operatorname{cosec}^2\theta \left(1 + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}\right)} \\ &= \frac{\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(1 + \sin^2\theta \times \cot^2\theta\right)} \\ &= \frac{(1 - \tan^2\theta)}{\left(1 + \sin^2\theta \times \frac{1}{\tan^2\theta}\right)} \\ &= \frac{1 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)} = \frac{1 - \frac{1}{2}}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)} \\ &= \frac{1/2}{1 + \frac{2}{3}} = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10} \end{aligned}$$

4. Question

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1 - \cos\theta}{1 + \cos\theta}$

Answer

$$\tan\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$\text{Now, } \frac{1 - \cos\theta}{1 + \cos\theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{1/5}{9/5} = \frac{1}{9} \frac{1 - \cos\theta}{1 + \cos\theta}$$

5. Question

If $\tan \theta = \frac{12}{5}$, find the value of $\frac{1 + \sin \theta}{1 - \sin \theta}$

Answer

$$\begin{aligned}
 \frac{1 + \sin \theta}{1 - \sin \theta} &= \frac{1 + \sin \theta}{1 - \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta} \\
 &= \frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} \\
 &= \frac{(1 + \sin \theta)^2}{\cos^2 \theta} \\
 &= \sec^2 \theta + \tan^2 \theta \\
 &= \tan^2 \theta + 1 + \tan^2 \theta \\
 &= 2\tan^2 \theta + 1 \\
 &= 2 \times \left(\frac{12}{5}\right)^2 + 1 \quad \left(\because \tan \theta = \frac{12}{5}\right) \\
 &= \frac{288}{25} + 1 \\
 &= \frac{288 + 25}{25} = \frac{313}{25}
 \end{aligned}$$

6. Question

If $\cot \theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$

Answer

$$\text{Given, } \cot \theta = \frac{1}{\sqrt{3}}$$

Now,

$$\begin{aligned}
 \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} &= \frac{\sin^2 \theta}{1 + \cos^2 \theta} \\
 &= \frac{1}{\frac{1 + \cos^2 \theta}{\sin^2 \theta}} \\
 &= \frac{1}{\frac{1}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}} \\
 &= \frac{1}{\operatorname{cosec}^2 \theta + \cot^2 \theta} \\
 &= \frac{1}{1 + \cot^2 \theta + \cot^2 \theta} \\
 &= \frac{1}{1 + 2\cot^2 \theta} \\
 &= \frac{1}{1 + 2 \times \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{1 + 2 \times \frac{1}{3}} = \frac{1}{1 + \frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}
 \end{aligned}$$

7. Question

If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2 \sin^2 A + 3 \cot^2 A}{4(\tan^2 A - \cos^2 A)}$

Answer

$$\text{Given, } \operatorname{cosec} A = \sqrt{2} \Rightarrow \cot^2 A = \operatorname{cosec}^2 A - 1 = (\sqrt{2})^2 - 1 = 2 - 1 = 1$$

$$\text{and, } \tan^2 A = \frac{1}{\cot^2 A} = \frac{1}{1} = 1$$

$$\Rightarrow \sin^2 A = \frac{1}{\operatorname{cosec}^2 A} = \frac{1}{(\sqrt{2})^2} = \frac{1}{2}$$

$$\Rightarrow \cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

Now,

$$\frac{2 \sin^2 A + 3 \cot^2 A}{4(\tan^2 A - \cos^2 A)} = \frac{2 \times \frac{1}{2} + 3 \times 1}{4\left(1 - \frac{3}{4}\right)} = \frac{1 + 3}{4 \times \frac{1}{4}} = 4$$

8. Question

If $\cot \theta = \sqrt{3}$, find the value of $\frac{\operatorname{cosec}^2 \theta + \cot^2 \theta}{\operatorname{cosec}^2 \theta - \sec^2 \theta}$

Answer

Given, $\cot \theta = \sqrt{3}$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + (\sqrt{3})^2 = 1 + 3 = 4$$

$$\text{and } \cot \theta = \sqrt{3} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sec^2 \theta = 1 + \tan^2 \theta = 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

Now,

$$\frac{\operatorname{cosec}^2 \theta + \cot^2 \theta}{\operatorname{cosec}^2 \theta - \sec^2 \theta} = \frac{4 + 3}{4 - \frac{4}{3}} = \frac{7}{8/3} = \frac{21}{8}$$

9. Question

If $3 \cos \theta = 1$, find the value of $\frac{6 \sin^2 \theta + \tan^2 \theta}{4 \cos \theta}$

Answer

$$3 \cos \theta = 1 \Rightarrow \cos \theta = \frac{1}{3}$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \sin \theta = \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$$

$$\frac{6 \sin^2 \theta + \tan^2 \theta}{4 \cos \theta} = \frac{6 \times \frac{8}{9} + 8}{4 \times \frac{1}{3}} = \frac{\frac{16}{3} + 8}{\frac{4}{3}} = \frac{\frac{30}{3}}{\frac{4}{3}} = \frac{30}{4} = 7 \frac{1}{2}$$

10. Question

If $\sqrt{3} \tan \theta = 3 \sin \theta$, find the value of $\sin^2 \theta - \cos^2 \theta$

Answer

$$\sqrt{3} \tan \theta = 3 \sin \theta$$

$$\sqrt{3} \frac{\sin \theta}{\cos \theta} = \sqrt{3} \times \sqrt{3} \sin \theta$$

$$\frac{1}{\cos \theta} = \sqrt{3}$$

$$\cos \theta = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \text{Now, } \sin^2 \theta - \cos^2 \theta &= 1 - \cos^2 \theta - \cos^2 \theta \\ &= 1 - 2 \cos^2 \theta \\ &= 1 - 2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \\ &= 1 - 2 \times \frac{1}{3} = \frac{1}{3} \end{aligned}$$

11. Question

If $\operatorname{cosec} \theta = \frac{13}{12}$, find the value of $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$

Answer

$$\operatorname{cosec} \theta = \frac{13}{12} \Rightarrow \sin \theta = \frac{12}{13} \text{ and } \cos \theta = \frac{5}{13}$$

$$\begin{aligned} \text{now, } \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} &= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}} \\ &= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} \\ &= \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13} \times \frac{13}{3} = 3 \end{aligned}$$

$$\operatorname{cosec} \theta = \frac{13}{12}$$

12. Question

If $\sin \theta + \cos \theta = \sqrt{2} \cos(90^\circ - \theta)$, find $\cot \theta$.

Answer

$$\begin{aligned} \sin \theta + \cos \theta &= \sqrt{2} \cos(90^\circ - \theta) \\ \cos \theta &= \sqrt{2} \sin \theta - \sin \theta \\ \cos \theta &= (\sqrt{2} - 1) \sin \theta \\ \frac{\cos \theta}{\sin \theta} &= \sqrt{2} - 1 \end{aligned}$$

CCE - Formative Assessment

1. Question

Define an identity.

Answer

An equation that is true for all values of the variables involved is said to be an identity. For example:

$$a^2 - b^2 = (a - b)(a + b)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

2. Question

What is the value of $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta$?

Answer

To find: $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta$

$$\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\therefore \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow (1 - \cos^2 \theta) \operatorname{cosec}^2 \theta = (1 - \cos^2 \theta) \frac{1}{\sin^2 \theta} \dots\dots\dots(i)$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta = 1 - \cos^2 \theta$$

$\Rightarrow$ from (i), we have

$$(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta = \sin^2 \theta \frac{1}{\sin^2 \theta} = 1$$

3. Question

What is the value of $(1 + \cot^2 \theta) \sin^2 \theta$?

Answer

To find: $(1 + \cot^2 \theta) \sin^2 \theta$

$$\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\therefore (1 + \cot^2 \theta) \sin^2 \theta = \operatorname{cosec}^2 \theta \sin^2 \theta$$

$$\text{Also, } \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow (1 + \cot^2 \theta) \sin^2 \theta = \operatorname{cosec}^2 \theta \sin^2 \theta = \frac{1}{\sin^2 \theta} \sin^2 \theta = 1$$

4. Question

What is the value of $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$?

Answer

To find: $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

$$\because 1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta}$$

$$\text{Also, we know that } \cos \theta = \frac{1}{\sec \theta}$$

$$\Rightarrow \cos^2 \theta = \frac{1}{\sec^2 \theta}$$

$$\Rightarrow \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta} = \sin^2 \theta + \cos^2 \theta$$

Also,

$$\because \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta} = \sin^2 \theta + \cos^2 \theta = 1$$

5. Question

If $\sec \theta + \tan \theta = x$, write the value of $\sec \theta - \tan \theta$ in terms of x .

Answer

Given: $\sec \theta + \tan \theta = x$ (i)

To find: $\sec \theta - \tan \theta$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta$$

$$\text{Now, } \because a^2 - b^2 = (a - b)(a + b)$$

$$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)$$

$\Rightarrow$ From (i), we have

$$1 = (\sec \theta - \tan \theta) x$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{x}$$

6. Question

If $\operatorname{cosec} \theta - \cot \theta = a$, write the value of $\operatorname{cosec} \theta + \cot \theta$.

Answer

Given: $\operatorname{cosec} \theta - \cot \theta = \alpha$ (i)

To find: $\operatorname{cosec} \theta + \cot \theta$

We know that $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\Rightarrow 1 = \operatorname{cosec}^2 \theta - \cot^2 \theta$$

$$\text{Now, } \because a^2 - b^2 = (a - b)(a + b)$$

$$\Rightarrow 1 = \operatorname{cosec}^2 \theta - \cot^2 \theta = (\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta)$$

$\Rightarrow$ From (i), we have

$$1 = \alpha (\operatorname{cosec} \theta + \cot \theta)$$

$$\Rightarrow \operatorname{cosec} \theta + \cot \theta = \frac{1}{\alpha}$$

7. Question

Write the value of $\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta$.

Answer

To find: $\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta$

$$\because \operatorname{cosec} (90^\circ - \theta) = \sec \theta$$

$$\therefore \operatorname{cosec}^2 (90^\circ - \theta) = \sec^2 \theta$$

$$\Rightarrow \operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta = \sec^2 \theta - \tan^2 \theta$$

$$\text{Now, } \because 1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore \operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta = \sec^2 \theta - \tan^2 \theta$$

$$= 1 + \tan^2 \theta - \tan^2 \theta = 1$$

8. Question

Write the value of $\sin A \cos (90^\circ - A) + \cos A \sin (90^\circ - A)$.

Answer

To find: $\sin A \cos (90^\circ - A) + \cos A \sin (90^\circ - A)$

$$\because \cos (90^\circ - A) = \sin A \text{ and } \sin (90^\circ - A) = \cos A \text{(i)}$$

$$\therefore \sin A \cos (90^\circ - A) + \cos A \sin (90^\circ - A)$$

$$= \sin A \sin A + \cos A \cos A \text{ [Using (i)]}$$

$$= \sin^2 A + \cos^2 A$$

$$\text{Now, } \because \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin A \cos (90^\circ - A) + \cos A \sin (90^\circ - A)$$

$$= \sin^2 A + \cos^2 A = 1$$

9. Question

Write the value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$.

Answer

To find: $\cot^2 \theta - \frac{1}{\sin^2 \theta}$

$$\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow \cot^2 \theta - \frac{1}{\sin^2 \theta} = \cot^2 \theta - \operatorname{cosec}^2 \theta$$

Also, we know that $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\Rightarrow \cot^2 \theta - \operatorname{cosec}^2 \theta = -1$$

$$\Rightarrow \cot^2 \theta - \frac{1}{\sin^2 \theta} = \cot^2 \theta - \operatorname{cosec}^2 \theta = -1$$

10. Question

If $x = a \sin \theta$ and $y = b \cos \theta$, what is the value of $b^2 x^2 + a^2 y^2$?

Answer

Given: $x = a \sin \theta$ and $y = b \cos \theta$

$$\Rightarrow x^2 = a^2 \sin^2 \theta \text{ and } y^2 = b^2 \cos^2 \theta \dots\dots\dots(i)$$

To find: $b^2 x^2 + a^2 y^2$

$$\text{Consider } b^2 x^2 + a^2 y^2 = b^2 a^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$$

$$= a^2 b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 b^2 (1) [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= a^2 b^2$$

11. Question

If $\sin \theta = \frac{4}{5}$, what is the value of $\cot \theta + \operatorname{cosec} \theta$?

Answer

$$\text{Given: } \sin \theta = \frac{4}{5}$$

To find: $\cot \theta + \operatorname{cosec} \theta$

$$\because \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\Rightarrow \cos \theta = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\text{Now, as } \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\text{Also, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{4/5} = \frac{5}{4}$$

$$\Rightarrow \cot \theta + \operatorname{cosec} \theta = \frac{3}{4} + \frac{5}{4} = \frac{3 + 5}{4} = \frac{8}{4} = 2$$

12. Question

What is the value of $9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta$?

Answer

To find: $9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta$

$$\text{Consider } 9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta = 9 (\cot^2 \theta - \operatorname{cosec}^2 \theta)$$

$$\text{Now } \because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow \cot^2 \theta - \operatorname{cosec}^2 \theta = -1$$

$$\Rightarrow 9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta = 9 (\cot^2 \theta - \operatorname{cosec}^2 \theta) = 9 (-1) = -9$$

13. Question

$$\text{What is the value of } 6 \tan^2 \theta - \frac{6}{\cos^2 \theta} ?$$

Answer

$$\text{To find: } 6 \tan^2 \theta - \frac{6}{\cos^2 \theta}$$

$$\because \sec \theta = \frac{1}{\cos \theta}$$

$$\Rightarrow \sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\Rightarrow 6 \tan^2 \theta - \frac{6}{\cos^2 \theta} = 6 \tan^2 \theta - 6 \sec^2 \theta = 6 (\tan^2 \theta - \sec^2 \theta)$$

$$\text{Now, as } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow \tan^2 \theta - \sec^2 \theta = -1$$

$$\Rightarrow 6 \tan^2 \theta - \frac{6}{\cos^2 \theta} = 6 (\tan^2 \theta - \sec^2 \theta) = 6 (-1) = -6$$

14. Question

$$\text{What is the value of } \frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \operatorname{cosec}^2 \theta} ?$$

Answer

$$\text{To find: } \frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \operatorname{cosec}^2 \theta}$$

$$\text{We know that } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\text{And } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow \tan^2 \theta - \sec^2 \theta = -1$$

$$\text{And } \cot^2 \theta - \operatorname{cosec}^2 \theta = -1$$

$$\Rightarrow \frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \operatorname{cosec}^2 \theta} = \frac{-1}{-1} = 1$$

15. Question

$$\text{What is the value of } (1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)?$$

Answer

$$\text{To find: } (1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$$

$$\because (a - b) (a + b) = a^2 - b^2$$

$$\therefore (1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$$

$$= (1 + \tan^2 \theta) (1 - \sin^2 \theta)$$

$$\text{Now, as } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow 1 - \sin^2 \theta = \cos^2 \theta \dots\dots\dots(i)$$

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$ (ii)

Using (i) and (ii), we have

$$(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$$

$$= (1 + \tan^2 \theta) (1 - \sin^2 \theta)$$

$$= \sec^2 \theta \cos^2 \theta$$

$$\therefore \sec \theta = \frac{1}{\cos \theta}$$

$$\Rightarrow \sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\Rightarrow (1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$$

$$= \sec^2 \theta \cos^2 \theta$$

$$= \frac{1}{\cos^2 \theta} \cos^2 \theta = 1$$

16. Question

If $\cos A = \frac{7}{25}$, find the value of $\tan A + \cot A$.

Answer

Given: $\cos A = \frac{7}{25}$

To find: $\tan A + \cot A$

$$\therefore \sin^2 A + \cos^2 A = 1$$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \left(\frac{7}{25}\right)^2} = \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{625 - 49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

Now, as $\tan A = \frac{\sin A}{\cos A} = \frac{24/25}{7/25} = \frac{24}{7}$

And $\cot A = \frac{1}{\tan A} = \frac{7}{24}$

$$\Rightarrow \tan A + \cot A = \frac{24}{7} + \frac{7}{24} = \frac{576 + 49}{168} = \frac{625}{168}$$

17. Question

If $\sin \theta = \frac{1}{3}$, then find the value of $2 \cot^2 \theta + 2$.

Answer

Given: $\sin \theta = \frac{1}{3}$

To find: The value of $2 \cot^2 \theta + 2$.

Solution: $\sin \theta = \frac{1}{3}$

$$\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{1/3} = 3$$

$$\Rightarrow \operatorname{cosec}^2 \theta = 3^2 = 9$$

Also, $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 = 9 - 1 = 8$$

$$\Rightarrow 2 \cot^2 \theta + 2 = 2(8) + 2 = 16 + 2 = 18$$

Hence, the value of $2 \cot^2 \theta + 2$ is 18.

18. Question

If $\cos \theta = \frac{3}{4}$, then find the value of $9 \tan^2 \theta + 9$.

Answer

Given: $\cos \theta = \frac{3}{4}$

To find: $9 \tan^2 \theta + 9$

$$\therefore \sec \theta = \frac{1}{\cos \theta} = \frac{4}{3}$$

$$\therefore \sec^2 \theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow \tan^2 \theta = \sec^2 \theta - 1 = \frac{16}{9} - 1 = \frac{16 - 9}{9} = \frac{7}{9}$$

$$\Rightarrow 9 \tan^2 \theta + 9 = 9 \left(\frac{7}{9}\right) + 9 = 7 + 9 = 16$$

19. Question

If $\sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = k$, then find the value of k .

Answer

Given: $\sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = k$

To find: k

Consider $\sec^2 \theta (1 + \sin \theta) (1 - \sin \theta)$

$$\therefore (a - b) (a + b) = a^2 - b^2$$

$$\therefore \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$$

Now, as $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$$

$$= \sec^2 \theta \cos^2 \theta$$

Now, $\therefore \sec \theta = \frac{1}{\cos \theta}$

$$\Rightarrow \sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$$

$$= \sec^2 \theta \cos^2 \theta$$

$$= \frac{1}{\cos^2 \theta} \cos^2 \theta = 1$$

$$\Rightarrow k = 1$$

20. Question

If $\operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta) = \lambda$, then find the value of λ .

Answer

$$\text{Given: } \operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta) = \lambda$$

To find: λ

$$\text{Consider } \operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta)$$

$$\because (a - b) (a + b) = a^2 - b^2$$

$$\therefore \operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta) = \operatorname{cosec}^2 \theta (1 - \cos^2 \theta)$$

$$\text{Now, as } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta) = \operatorname{cosec}^2 \theta (1 - \cos^2 \theta)$$

$$= \operatorname{cosec}^2 \theta \sin^2 \theta$$

$$\text{Now, } \because \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta (1 + \cos \theta) (1 - \cos \theta) = \operatorname{cosec}^2 \theta (1 - \cos^2 \theta)$$

$$= \operatorname{cosec}^2 \theta \sin^2 \theta$$

$$= \frac{1}{\sin^2 \theta} \sin^2 \theta = 1$$

21. Question

If $\sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta) (1 + \cot^2 \theta) = \lambda$, then find the value of λ .

Answer

$$\text{Given: } \sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta) (1 + \cot^2 \theta) = \lambda$$

To find: λ

$$\text{We know that } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\text{And } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow \sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta) (1 + \cot^2 \theta)$$

$$= \sin^2 \theta \cos^2 \theta \sec^2 \theta \operatorname{cosec}^2 \theta$$

$$\text{Now, } \because \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\text{And } \because \sec \theta = \frac{1}{\cos \theta}$$

$$\Rightarrow \sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\Rightarrow \sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta) (1 + \cot^2 \theta)$$

$$= \sin^2 \theta \cos^2 \theta \sec^2 \theta \operatorname{cosec}^2 \theta$$

$$= \sin^2 \theta \cos^2 \theta \frac{1}{\cos^2 \theta} \frac{1}{\sin^2 \theta} = 1$$

$$\Rightarrow \lambda = 1$$

22. Question

If $5x = \sec \theta$ and $\frac{5}{x} = \tan \theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Answer

Given: $5x = \sec \theta$

$$\Rightarrow x = \frac{\sec \theta}{5}$$

$$\Rightarrow x^2 = \frac{\sec^2 \theta}{25} \dots\dots\dots(i)$$

And $\frac{5}{x} = \tan \theta$

$$\Rightarrow x = \frac{5}{\tan \theta}$$

$$\Rightarrow x^2 = \frac{25}{\tan^2 \theta}$$

$$\Rightarrow \frac{1}{x^2} = \frac{\tan^2 \theta}{25} \dots\dots\dots(ii)$$

To find: $5\left(x^2 - \frac{1}{x^2}\right)$

$$\text{Consider } 5\left(x^2 - \frac{1}{x^2}\right) = 5\left(\frac{\sec^2 \theta}{25} - \frac{1}{x^2}\right) \text{ [Using (i)]}$$

$$= 5\left(\frac{\sec^2 \theta}{25} - \frac{\tan^2 \theta}{25}\right) \text{ [Using (ii)]}$$

$$= 5\left(\frac{\sec^2 \theta - \tan^2 \theta}{25}\right) = \frac{1}{5}(\sec^2 \theta - \tan^2 \theta)$$

Now, as $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta$$

$$\Rightarrow 5\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{5}(\sec^2 \theta - \tan^2 \theta) = \frac{1}{5}$$

23. Question

If $\operatorname{cosec} \theta = 2x$ and $\cot \theta = \frac{2}{x}$, find the value of $2\left(x^2 - \frac{1}{x^2}\right)$

Answer

Given: $\operatorname{cosec} \theta = 2x$

$$\Rightarrow x = \frac{\operatorname{cosec} \theta}{2}$$

$$\Rightarrow x^2 = \frac{\operatorname{cosec}^2 \theta}{4} \dots\dots\dots(i)$$

And $\cot \theta = \frac{2}{x}$

$$\Rightarrow x = \frac{2}{\cot \theta}$$

$$\Rightarrow x^2 = \frac{4}{\cot^2 \theta}$$

$$\Rightarrow \frac{1}{x^2} = \frac{\cot^2 \theta}{4} \dots\dots\dots(ii)$$

To find: $2\left(x^2 - \frac{1}{x^2}\right)$

$$\text{Consider } 2 \left(x^2 - \frac{1}{x^2} \right) = 2 \left(\frac{\operatorname{cosec}^2 \theta}{4} - \frac{1}{x^2} \right) \text{ [Using (i)]}$$

$$= 2 \left(\frac{\operatorname{cosec}^2 \theta}{4} - \frac{\cot^2 \theta}{4} \right) \text{ [Using (ii)]}$$

$$= 2 \left(\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{4} \right) = \frac{1}{2} (\operatorname{cosec}^2 \theta - \cot^2 \theta)$$

$$\text{Now, as } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow 1 = \operatorname{cosec}^2 \theta - \cot^2 \theta$$

$$\Rightarrow 2 \left(x^2 - \frac{1}{x^2} \right) = \frac{1}{2} (\operatorname{cosec}^2 \theta - \cot^2 \theta) = \frac{1}{2}$$

1. Question

If $\sec \theta + \tan \theta = x$, then $\sec \theta =$

A. $\frac{x^2 + 1}{x}$

B. $\frac{x^2 + 1}{2x}$

C. $\frac{x^2 - 1}{2x}$

D. $\frac{x^2 - 1}{x}$

Answer

Given: $\sec \theta + \tan \theta = x$ (i)

To find: $\sec \theta$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

$$\because a^2 - b^2 = (a - b)(a + b)$$

$$\therefore \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$\Rightarrow$ From (i), we have

$$\Rightarrow (\sec \theta - \tan \theta) x = 1$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{x} \text{(ii)}$$

Adding (i) and (ii), we get

$$\sec \theta + \sec \theta = x + \frac{1}{x}$$

$$\Rightarrow \sec \theta = \frac{x^2 + 1}{2x}$$

$$\Rightarrow \sec \theta = \frac{x^2 + 1}{2x}$$

2. Question

If $\sec \theta + \tan \theta = x$, then $\tan \theta =$

A. $\frac{x^2 + 1}{x}$

B. $\frac{x^2 - 1}{x}$

C. $\frac{x^2 + 1}{2x}$

D. $\frac{x^2 - 1}{2x}$

Answer

Given: $\sec \theta + \tan \theta = x$ (i)

To find: $\tan \theta$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

$$\therefore a^2 - b^2 = (a - b)(a + b)$$

$$\therefore \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$\Rightarrow$ From (i), we have

$$\Rightarrow (\sec \theta - \tan \theta) x = 1$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{x} \text{(ii)}$$

Subtracting (ii) from (i), we get

$$\tan \theta + \tan \theta = x - \frac{1}{x}$$

$$\Rightarrow 2 \tan \theta = \frac{x^2 - 1}{x}$$

$$\Rightarrow \tan \theta = \frac{x^2 - 1}{2x}$$

3. Question

$\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$ is equal to

A. $\sec \theta + \tan \theta$

B. $\sec \theta - \tan \theta$

C. $\sec^2 \theta + \tan^2 \theta$

D. $\sec^2 \theta - \tan^2 \theta$

Answer

Note: Since all the options involve the trigonometric ratios $\sec \theta$ and $\tan \theta$, so we divide the whole term (numerator as well as denominator) by $\cos \theta$.

To find: $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$

Consider $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$

Dividing numerator and denominator by $\cos \theta$, we get

$$\begin{aligned}\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} &= \sqrt{\frac{\frac{1 + \sin \theta}{\cos \theta}}{\frac{1 - \sin \theta}{\cos \theta}}} = \sqrt{\frac{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}} \\ &= \sqrt{\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta}} \left[\begin{array}{l} \because \sec \theta = \frac{1}{\cos \theta} \\ \text{and } \tan \theta = \frac{\sin \theta}{\cos \theta} \end{array} \right]\end{aligned}$$

Rationalizing the term by multiplying it by $\sqrt{\sec \theta + \tan \theta}$,

$$\begin{aligned}\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} &= \sqrt{\frac{\frac{1 + \sin \theta}{\cos \theta}}{\frac{1 - \sin \theta}{\cos \theta}}} = \sqrt{\frac{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}} = \sqrt{\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta}} \\ &= \sqrt{\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta}} \times \sqrt{\frac{\sec \theta + \tan \theta}{\sec \theta + \tan \theta}} \\ &= \sqrt{\frac{(\sec \theta + \tan \theta)^2}{\sec^2 \theta - \tan^2 \theta}}\end{aligned}$$

Now, as $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \sqrt{\frac{(\sec \theta + \tan \theta)^2}{\sec^2 \theta - \tan^2 \theta}} = \sqrt{(\sec \theta + \tan \theta)^2} = \sec \theta + \tan \theta$$

4. Question

The value of $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$ is

- A. $\cot \theta - \operatorname{cosec} \theta$
- B. $\operatorname{cosec} \theta + \cot \theta$
- C. $\operatorname{cosec}^2 \theta + \cot^2 \theta$
- D. $(\cot \theta + \operatorname{cosec} \theta)^2$

Answer

Note: Since all the options involve the trigonometric ratios $\operatorname{cosec} \theta$ and $\cot \theta$, so we divide the whole term (numerator as well as denominator) by $\sin \theta$.

To find: $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$

Consider $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$

Dividing numerator and denominator by $\sin \theta$, we get

$$\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \sqrt{\frac{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}} = \sqrt{\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta}} \left[\begin{array}{l} \because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \\ \text{and } \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right]$$

Rationalizing the term by multiplying it by $\sqrt{\operatorname{cosec} \theta + \cot \theta}$,

$$\begin{aligned}
 \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} &= \sqrt{\frac{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}} = \sqrt{\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta}} \\
 &= \sqrt{\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta}} \times \sqrt{\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta + \cot \theta}} \\
 &= \sqrt{\frac{(\operatorname{cosec} \theta + \cot \theta)^2}{\operatorname{cosec}^2 \theta - \cot^2 \theta}}
 \end{aligned}$$

Now, as $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\begin{aligned}
 \Rightarrow \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} &= \sqrt{\frac{(\operatorname{cosec} \theta + \cot \theta)^2}{\operatorname{cosec}^2 \theta - \cot^2 \theta}} = \sqrt{(\operatorname{cosec} \theta + \cot \theta)^2} \\
 &= \operatorname{cosec} \theta + \cot \theta
 \end{aligned}$$

5. Question

$\sec^4 A - \sec^2 A$ is equal to

- A. $\tan^2 A - \tan^4 A$
- B. $\tan^4 A - \tan^2 A$
- C. $\tan^4 A + \tan^2 A$
- D. $\tan^2 A + \tan^4 A$

Answer

Note: Since all the options involve the trigonometric ratio $\tan \theta$, so we use the identity $1 + \tan^2 \theta = \sec^2 \theta$.

To find: $\sec^4 A - \sec^2 A$

$$\text{Consider } \sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$$

Now, as $\sec^2 A = 1 + \tan^2 A$

$$\Rightarrow \sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$$

$$= (1 + \tan^2 A)^2 - (1 + \tan^2 A)$$

$$= 1 + \tan^4 A + 2 \tan^2 A - 1 - \tan^2 A$$

$$= \tan^4 A + \tan^2 A$$

6. Question

$\cos^4 A - \sin^4 A$ is equal to

- A. $2 \cos^2 A + 1$
- B. $2 \cos^2 A - 1$
- C. $2 \sin^2 A - 1$
- D. $2 \sin^2 A + 1$

Answer

To find: $\cos^4 A - \sin^4 A$

$$\text{Consider } \cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$\therefore a^2 - b^2 = (a - b)(a + b)$$

$$\begin{aligned}
 \therefore \cos^4 A - \sin^4 A &= (\cos^2 A)^2 - (\sin^2 A)^2 \\
 &= (\cos^2 A - \sin^2 A) (\cos^2 A + \sin^2 A) \\
 &= (\cos^2 A - \sin^2 A) [\because \cos^2 A + \sin^2 A = 1] \\
 &= \cos^2 A - (1 - \cos^2 A) [\because \sin^2 A = 1 - \cos^2 A] \\
 &= \cos^2 A - 1 + \cos^2 A = 2 \cos^2 A - 1
 \end{aligned}$$

7. Question

$\frac{\sin \theta}{1 + \cos \theta}$ is equal to

- A. $\frac{1 + \cos \theta}{\sin \theta}$
- B. $\frac{1 - \cos \theta}{\cos \theta}$
- C. $\frac{1 - \cos \theta}{\sin \theta}$
- D. $\frac{1 - \sin \theta}{\cos \theta}$

Answer

To find: $\frac{\sin \theta}{1 + \cos \theta}$

Consider $\frac{\sin \theta}{1 + \cos \theta}$

Rationalizing the above fraction by $(1 - \cos \theta)$,

$$\begin{aligned}
 \frac{\sin \theta}{1 + \cos \theta} &= \frac{\sin \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta} = \frac{\sin \theta (1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &= \frac{\sin \theta (1 - \cos \theta)}{(1 - \cos^2 \theta)} [\because (a - b)(a + b) = a^2 - b^2]
 \end{aligned}$$

$$\because \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \frac{\sin \theta}{1 + \cos \theta} = \frac{\sin \theta (1 - \cos \theta)}{(1 - \cos^2 \theta)} = \frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

8. Question

$\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta}$ is equal to

- A. 0
- B. 1
- C. $\sin \theta + \cos \theta$
- D. $\sin \theta - \cos \theta$

Answer

Given: $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta}$

To find: The value of $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta}$

Solution:

Use:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

So,

$$\begin{aligned} \frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} &= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} = \frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} - \frac{\cos^2 \theta}{\sin \theta - \cos \theta} \\ &= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \end{aligned}$$

Using the identity,

$$a^2 - b^2 = (a - b)(a + b)$$

$$\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} = \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} = \frac{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta$$

9. Question

The value of $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$ is

- A. 1
- B. 2
- C. 4
- D. 0

Answer

To find: $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$

Consider $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$

$$\begin{aligned} &= \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \\ &\left[\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}, \sec \theta = \frac{1}{\cos \theta}\right] \\ &= \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \\ &= \left(\frac{(\sin \theta + \cos \theta) - 1}{\sin \theta}\right) \left(\frac{(\sin \theta + \cos \theta) + 1}{\cos \theta}\right) \\ &= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta} [\because (a - b)(a + b) = a^2 - b^2] \\ &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2 \end{aligned}$$

10. Question

$\frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1}$ is equal to

- A. $2 \tan \theta$
- B. $2 \sec \theta$
- C. $2 \operatorname{cosec} \theta$
- D. $2 \tan \theta \sec \theta$

Answer

To find: $\frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1}$

Consider $\frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1}$

$$\because \tan \theta = \frac{\sin \theta}{\cos \theta}, \sec \theta = \frac{1}{\cos \theta}$$

$$\therefore \frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} - 1} + \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} + 1} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1 - \cos \theta}{\cos \theta}} + \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1 + \cos \theta}{\cos \theta}}$$

$$= \frac{\sin \theta}{1 - \cos \theta} + \frac{\sin \theta}{1 + \cos \theta}$$

$$= \frac{\sin \theta (1 + \cos \theta) + \sin \theta (1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$= \frac{\sin \theta + \sin \theta \cos \theta + \sin \theta - \sin \theta \cos \theta}{(1 - \cos^2 \theta)}$$

$$= \frac{2 \sin \theta}{\sin^2 \theta} [\because \sin^2 \theta = 1 - \cos^2 \theta]$$

$$= \frac{2}{\sin \theta}$$

$$= 2 \operatorname{cosec} \theta \left[\because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right]$$

11. Question

$(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$ is equal

- A. 0
- B. 1
- C. -1
- D. None of these

Answer

To find: $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$

$$\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\therefore (\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$$

$$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \right)$$

Now, as $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\text{And } \cos^2 \theta = 1 - \sin^2 \theta$$

$$\begin{aligned} \Rightarrow \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \right) \\ = \left(\frac{\cos^2 \theta}{\sin \theta} \right) \left(\frac{\sin^2 \theta}{\cos \theta} \right) \left(\frac{1}{\sin \theta \cos \theta} \right) = 1 \end{aligned}$$

Hence the answer is 'B'

12. Question

If $x = a \cos \theta$ and $y = b \sin \theta$, then $b^2 x^2 + a^2 y^2 =$

A. $a^2 b^2$

B. ab

C. $a^4 b^4$

D. $a^2 + b^2$

Answer

Given: $x = a \sin \theta$ and $y = b \cos \theta$

$$\Rightarrow x^2 = a^2 \sin^2 \theta \text{ and } y^2 = b^2 \cos^2 \theta \dots\dots\dots(i)$$

To find: $b^2 x^2 + a^2 y^2$

$$\text{Consider } b^2 x^2 + a^2 y^2 = b^2 a^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$$

$$= a^2 b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 b^2 (1) [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= a^2 b^2$$

13. Question

If $x = a \sec \theta$ and $y = b \tan \theta$, then $b^2 x^2 - a^2 y^2 =$

A. ab

B. $a^2 - b^2$

C. $a^2 + b^2$

D. $a^2 b^2$

Answer

Given: $x = a \sec \theta$ and $y = b \tan \theta$

$$\Rightarrow x^2 = a^2 \sec^2 \theta \text{ and } y^2 = b^2 \tan^2 \theta \dots\dots\dots(i)$$

To find: $b^2 x^2 - a^2 y^2$

$$\text{Consider } b^2 x^2 - a^2 y^2 = b^2 a^2 \sec^2 \theta - a^2 b^2 \tan^2 \theta$$

$$= a^2 b^2 (\sec^2 \theta - \tan^2 \theta)$$

$$= a^2 b^2 (1) [\because \sec^2 \theta - \tan^2 \theta = 1]$$

$$= a^2 b^2$$

14. Question

$$\frac{\cot \theta}{\cot \theta - \cot 3\theta} + \frac{\tan \theta}{\tan \theta - \tan 3\theta} \text{ is equal to}$$

- A. 0
- B. 1
- C. -1
- D. 2

Answer

To find: $\frac{\cot \theta}{\cot \theta - \cot 3\theta} + \frac{\tan \theta}{\tan \theta - \tan 3\theta}$

Consider $\frac{\cot \theta}{\cot \theta - \cot 3\theta} + \frac{\tan \theta}{\tan \theta - \tan 3\theta}$

$$= \frac{\cot \theta (\tan \theta - \tan 3\theta) + \tan \theta (\cot \theta - \cot 3\theta)}{(\cot \theta - \cot 3\theta)(\tan \theta - \tan 3\theta)}$$

$$= \frac{\cot \theta \tan \theta - \cot \theta \tan 3\theta + \tan \theta \cot \theta - \tan \theta \cot 3\theta}{\cot \theta \tan \theta - \cot \theta \tan 3\theta - \cot 3\theta \tan \theta + \cot 3\theta \tan 3\theta}$$

$$= \frac{1 - \cot \theta \tan 3\theta + 1 - \tan \theta \cot 3\theta}{1 - \cot \theta \tan 3\theta - \cot 3\theta \tan \theta + 1} \left[\because \cot \theta = \frac{1}{\tan \theta} \Rightarrow \cot \theta \tan \theta = 1 \right]$$

$$= \frac{2 - \cot \theta \tan 3\theta - \tan \theta \cot 3\theta}{2 - \cot \theta \tan 3\theta - \tan \theta \cot 3\theta} = 1$$

15. Question

$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$ is equal to

- A. 0
- B. 1
- C. -1
- D. None of these

Answer

To find: $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$

First, we consider

$$\sin^6 \theta + \cos^6 \theta = (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$\text{Now, as } (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$\Rightarrow a^3 + b^3 = (a + b)^3 - 3a^2b - 3ab^2$$

$$\Rightarrow \sin^6 \theta + \cos^6 \theta$$

$$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta)^3 - 3(\sin^2 \theta)^2 \cos^2 \theta - 3 \sin^2 \theta (\cos^2 \theta)^2$$

$$= 1 - 3 \sin^4 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^4 \theta [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta [\because \sin^2 \theta + \cos^2 \theta = 1] \dots\dots\dots(i)$$

Next, we consider

$$\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta)^2 + (\cos^2 \theta)^2$$

$$\text{Now, as } (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow a^2 + b^2 = (a + b)^2 - 2ab$$

$$\Rightarrow \sin^4 \theta + \cos^4 \theta$$

$$= (\sin^2 \theta)^2 + (\cos^2 \theta)^2$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 2 \sin^2 \theta \cos^2 \theta \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \dots\dots\dots(ii)$$

Now, using (i) and (ii), we have

$$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$$

$$= 2(1 - 2 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta)$$

$$= 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta$$

$$= 2 - 3 = -1$$

16. Question

If $a \cos \theta + b \sin \theta = 4$ and $a \sin \theta - b \cos \theta = 3$, then $a^2 + b^2 =$

- A. 7
- B. 12
- C. 25
- D. None of these

Answer

Given: $a \cos \theta + b \sin \theta = 4$

Squaring both sides, we get

$$(a \cos \theta + b \sin \theta)^2 = 4^2$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = 16 \dots\dots\dots(i)$$

and $a \sin \theta - b \cos \theta = 3$

Squaring both sides, we get

$$(a \sin \theta - b \cos \theta)^2 = 3^2$$

$$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = 9 \dots\dots\dots(ii)$$

To find: $a^2 + b^2$

Adding (i) and (ii), we get

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta$$

$$+ a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = 16 + 9$$

$$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = 25$$

$$\Rightarrow a^2 + b^2 = 25 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

17. Question

If $a \cot \theta + b \operatorname{cosec} \theta = p$ and $b \cot \theta + a \operatorname{cosec} \theta = q$, then $p^2 - q^2 =$

- A. $a^2 - b^2$
- B. $b^2 - a^2$
- C. $a^2 + b^2$
- D. $b - a$

Answer

Given: $a \cot \theta + b \operatorname{cosec} \theta = p$

Squaring both sides, we get

$$(a \cot \theta + b \operatorname{cosec} \theta)^2 = p^2$$

$$\Rightarrow a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta = p^2 \dots\dots(i)$$

$$\text{and } b \cot \theta + a \operatorname{cosec} \theta = q$$

Squaring both sides, we get

$$(b \cot \theta + a \operatorname{cosec} \theta)^2 = q^2$$

$$\Rightarrow b^2 \cot^2 \theta + a^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta = q^2 \dots\dots(ii)$$

$$\text{To find: } p^2 - q^2$$

Subtracting (ii) from (i), we get

$$a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta - b^2 \cot^2 \theta - a^2 \operatorname{cosec}^2 \theta - 2ab \cot \theta \operatorname{cosec} \theta = p^2 - q^2$$

$$\Rightarrow p^2 - q^2 = a^2 (\cot^2 \theta - \operatorname{cosec}^2 \theta) + b^2 (\operatorname{cosec}^2 \theta - \cot^2 \theta)$$

$$= a^2 (-1) + b^2 (1) [\because \operatorname{cosec}^2 \theta - \cot^2 \theta]$$

$$= b^2 - a^2$$

18. Question

The value of $\sin^2 29^\circ + \sin^2 61^\circ$ is

- A. 1
- B. 0
- C. $2 \sin^2 29^\circ$
- D. $2 \cos^2 61^\circ$

Answer

To find: $\sin^2 29^\circ + \sin^2 61^\circ$

Consider $\sin^2 29^\circ + \sin^2 61^\circ$

$$\because 29 = 90 - 61$$

$$\therefore \sin^2 29^\circ + \sin^2 61^\circ = \sin^2 (90^\circ - 61^\circ) + \sin^2 61^\circ$$

Now, as $\sin (90^\circ - \theta) = \cos \theta$

$$\Rightarrow \sin^2 29^\circ + \sin^2 61^\circ = \sin^2 (90^\circ - 61^\circ) + \sin^2 61^\circ$$

$$= \cos^2 61^\circ + \sin^2 61^\circ$$

$$= 1 [\sin^2 \theta + \cos^2 \theta = 1]$$

19. Question

If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \theta$, then

- A. $x^2 + y^2 + z^2 = r^2$
- B. $x^2 + y^2 - z^2 = r^2$
- C. $x^2 - y^2 + z^2 = r^2$
- D. $z^2 + y^2 - x^2 = r^2$

Answer

Given: $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \theta$,

Solution: $x = r \sin \theta \cos \phi$

Squaring both sides, we get

$$x^2 = r^2 \sin^2 \theta \cos^2 \phi \dots\dots\dots(i)$$

$$\text{and } y = r \sin \theta \sin \phi$$

Squaring both sides, we get

$$\Rightarrow y^2 = r^2 \sin^2 \theta \sin^2 \phi \dots\dots\dots(ii)$$

$z = r \cos \theta$ Squaring both sides, we get

$$\Rightarrow z^2 = r^2 \cos^2 \theta \dots\dots\dots(iii)$$

Adding (i), (ii) and (iii), we get

$$x^2 + y^2 + z^2 = r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta$$

$$= r^2 (\sin^2 \theta \cos^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \theta)$$

$$= r^2 [\sin^2 \theta (\cos^2 \phi + \sin^2 \phi) + \cos^2 \theta]$$

$$\because \sin^2 \theta + \cos^2 \theta = 1$$

$$= r^2 [\sin^2 \theta + \cos^2 \theta]$$

Again apply the identity $\sin^2 \theta + \cos^2 \theta = 1$

$$= r^2 \text{ Hence } x^2 + y^2 + z^2 = r^2$$

20. Question

If $\sin \theta + \sin^2 = 1$, then $\cos^2 \theta + \cos^4 \theta =$

- A. -1
- B. 1
- C. 0
- D. None of these

Answer

Given: $\sin \theta + \sin^2 \theta = 1$

$$\Rightarrow \sin \theta = 1 - \sin^2 \theta = \cos^2 \theta [\because \sin^2 \theta + \cos^2 \theta = 1] \dots\dots(i)$$

$$\Rightarrow \sin^2 \theta = (\cos^2 \theta)^2 = \cos^4 \theta \dots\dots(ii)$$

To find: $\cos^2 \theta + \cos^4 \theta$

Consider $\cos^2 \theta + \cos^4 \theta = \sin \theta + \sin^2 \theta$ [Using (i) and (ii)]

$$= 1 \text{ [Given]}$$

21. Question

If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then $a^2 + b^2 =$

- A. $m^2 - n^2$
- B. $m^2 n^2$
- C. $n^2 - m^2$
- D. $m^2 + n^2$

Answer

Given: $a \cos \theta + b \sin \theta = m$

Squaring both sides, we get

$$(a \cos \theta + b \sin \theta)^2 = m^2$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta = m^2 \dots\dots(i)$$

And $a \sin \theta - b \cos \theta = n$

Squaring both sides, we get

$$(a \sin \theta - b \cos \theta)^2 = n^2$$

$$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = n^2 \dots\dots(ii)$$

$$\text{To find: } a^2 + b^2$$

Adding (i) and (ii), we get

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = m^2 + n^2$$

$$\Rightarrow a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$$

$$\Rightarrow a^2 + b^2 = m^2 + n^2 [\because \sin^2 \theta + \cos^2 \theta = 1]$$

22. Question

If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$

A. -1

B. 0

C. 1

D. None of these

Answer

$$\text{Given: } \cos A + \cos^2 A = 1$$

$$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A [\because \sin^2 A + \cos^2 A = 1] \dots\dots(i)$$

Squaring both sides, we get

$$\Rightarrow \cos^2 A = (\sin^2 A)^2 = \sin^4 A \dots\dots(ii)$$

$$\text{To find: } \sin^2 A + \sin^4 A$$

$$\text{Consider } \sin^2 A + \sin^4 A = \cos A + \cos^2 A [\text{From (i) and (ii)}]$$

$$= 1$$

23. Question

If $x = a \sec \theta \cos \phi$, $y = b \sec \theta \sin \phi$ and $z = c \tan \theta$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

A. $\frac{z^2}{c^2}$

B. $1 - \frac{z^2}{c^2}$

C. $\frac{z^2}{c^2} - 1$

D. $1 + \frac{z^2}{c^2}$

Answer

$$\text{Given: } x = a \sec \theta \cos \phi$$

Squaring both sides, we get

$$x^2 = a^2 \sec^2 \theta \cos^2 \phi$$

$$\text{and } y = b \sec \theta \sin \phi$$

Squaring both sides, we get

$$y^2 = b^2 \sec^2 \theta \sin^2 \phi$$

$$\text{And } z = c \tan \theta$$

$$\Rightarrow z^2 = c^2 \tan^2 \theta$$

$$\Rightarrow \tan^2 \theta = \frac{z^2}{c^2} \dots\dots(i)$$

$$\text{To find: } \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\text{Consider } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2 \sec^2 \theta \cos^2 \phi}{a^2} + \frac{b^2 \sec^2 \theta \sin^2 \phi}{b^2}$$

$$= \sec^2 \theta \cos^2 \phi + \sec^2 \theta \sin^2 \phi$$

$$= \sec^2 \theta (\cos^2 \phi + \sin^2 \phi)$$

$$= \sec^2 \theta [\because \sin^2 \phi + \cos^2 \phi = 1]$$

$$= 1 + \tan^2 \theta [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$= 1 + \frac{z^2}{c^2}$$

24. Question

If $a \cos \theta - b \sin \theta = c$, then $a \sin \theta + b \cos \theta =$

A. $\pm \sqrt{a^2 + b^2 + c^2}$

B. $\pm \sqrt{a^2 + b^2 - c^2}$

C. $\pm \sqrt{c^2 - a^2 + b^2}$

D. None of these

Answer

Given: $a \cos \theta - b \sin \theta = c$

To find: $a \sin \theta + b \cos \theta$

Consider $a \cos \theta - b \sin \theta = c$

Squaring both sides, we get

$$(a \cos \theta - b \sin \theta)^2 = c^2$$

$$\because (a - b)^2 = a^2 + b^2 - 2ab$$

$$\therefore a \cos \theta - b \sin \theta = c$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta = c^2 \dots\dots(i)$$

$$\text{Now, } \because \sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta = 1 - \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta$$

$\Rightarrow$ From (i), we have

$$\Rightarrow a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \sin \theta \cos \theta = c^2$$

$$\Rightarrow a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = c^2$$

$$\Rightarrow a^2 + b^2 - (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2$$

$$\Rightarrow -(a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2 - a^2 - b^2$$

$$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = a^2 + b^2 - c^2$$

$$\Rightarrow (a \sin \theta)^2 + (b \cos \theta)^2 + 2 (a \sin \theta) (b \cos \theta) = a^2 + b^2 - c^2$$

$$\Rightarrow (a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$$

25. Question

$9 \sec^2 A - 9 \tan^2 A$ is equal to

- A. 1
- B. 9
- C. 8
- D. 0

Answer

To find: $9 \sec^2 A - 9 \tan^2 A$

Consider $9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$$\because 1 + \tan^2 A = \sec^2 A$$

$$\therefore 9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$$

$$= 9 (1 + \tan^2 A - \tan^2 A) = 9$$

26. Question

$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta) =$

- A. 0
- B. 1
- C. 1
- D. -1

Answer

To find: $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

Consider $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$\left[\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}, \sec \theta = \frac{1}{\cos \theta}\right]$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \left(\frac{(\sin \theta + \cos \theta) + 1}{\cos \theta}\right) \left(\frac{(\sin \theta + \cos \theta) - 1}{\sin \theta}\right)$$

$$= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta} [\because (a - b)(a + b) = a^2 - b^2]$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

27. Question

$(\sec A + \tan A) (1 - \sin A) =$

- A. $\sec A$
- B. $\sin A$
- C. $\operatorname{cosec} A$
- D. $\cos A$

Answer

To find: $(\sec A + \tan A)(1 - \sin A)$

Consider $(\sec A + \tan A)(1 - \sin A)$

We know that $\sec A = \frac{1}{\cos A}$ and $\tan A = \frac{\sin A}{\cos A}$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$\because (a + b)(a - b) = a^2 - b^2$$

$$\therefore (\sec A + \tan A)(1 - \sin A) = \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A) = \frac{1 - \sin^2 A}{\cos A}$$

$$\text{Also, } \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

28. Question

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is equal to

- A. $\sec^2 A$
- B. -1
- C. $\cot^2 A$
- D. $\tan^2 A$

Answer

To find: $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

Consider $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$$\because 1 + \tan^2 A = \sec^2 A \text{ and } 1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} \left[\because \sec A = \frac{1}{\cos A} \text{ and } \operatorname{cosec} A = \frac{1}{\sin A} \right]$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \left[\because \tan A = \frac{\sin A}{\cos A} \right]$$