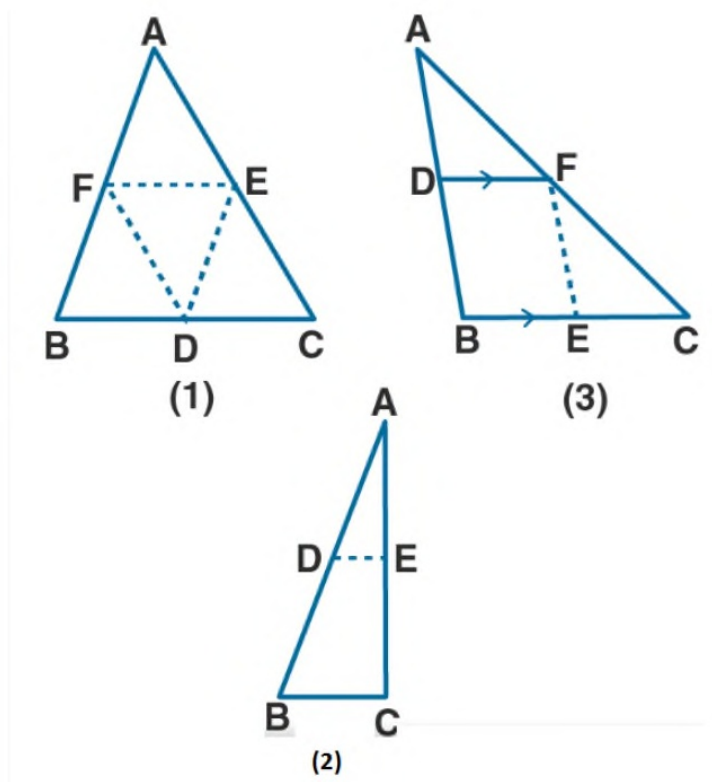


CHAPTER – 11
MID – POINT THEOREM

Exercise 11.1

- 1.**
 - (a)** In the figure (1) given below, D, E and F are mid – point of the sides BC, CA and AB respectively of $\triangle ABC$. If $AB = 6$ cm, $BC = 4.8$ cm and $CA = 5.6$ cm, find the perimeter of
 - (i)** The trapezium FBCE
 - (ii)** The triangle DEF.
 - (b)** In the figure (2) given below, D and E are mid-points of the side AB and AC respectively. If $BC = 5.6$ cm and $\angle B = 72^\circ$, compute
 - (i)** DE
 - (ii)** $\angle ADE$.
 - (c)** In the figure (3) given below, D and E are mid-point of AB, BC respectively and $DF \parallel BC$. Prove that DBEF is a parallelogram. Calculate AC if $AF = 2.6$ cm.

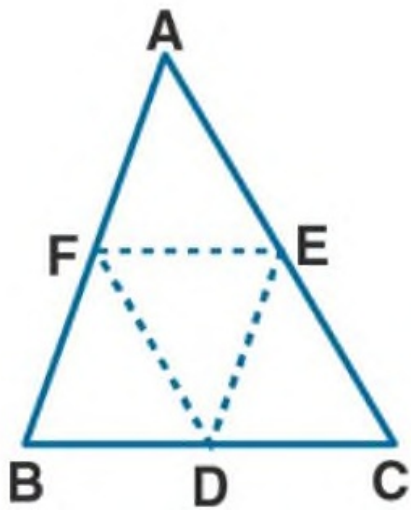


Solution:

(a)

(i) Given: $AB = 6$ cm, $BC = 4.8$ cm, and $CA = 5.6$ cm

To find: The perimeter of trapezium FBCA.



It is given that

F is the mid-point of AB

We know that

$$BF = \frac{1}{2} AB = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm} \quad \dots (1)$$

It is given that

E is the mid-point of AC

We know that

$$CE = \frac{1}{2} AC = \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm} \quad \dots (2)$$

Here F and E are the mid-point of AB and CA

$FE \parallel BC$

We know that

$$FE = \frac{1}{2} BC = \frac{1}{2} \times 4.8 = 2.4 \text{ cm} \quad \dots (3)$$

Here

Perimeter of trapezium FBCE = BF + BC + CE + EF

Now substituting the value from all the equations

$$= 3 + 4.8 + 2.8 + 2.4$$

$$= 13 \text{ cm}$$

Therefore, the perimeter of trapezium FBCE is 13 cm.

(ii) D, E and F are the midpoints of sides BC, CA and AB of $\triangle ABC$

Here $EF \parallel BC$

$$EF = \frac{1}{2} BC = \frac{1}{2} \times 4.8 = 2.4 \text{ cm}$$

$$DE = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm}$$

$$FD = \frac{1}{2} AC = \frac{1}{2} \times 5.6 = 2.8 \text{ cm}$$



We know that

$$\text{Perimeter of } \triangle DEF = DE + EF + FD$$

Substituting the values

$$= 3 + 2.4 + 2.8$$

$$= 8.2 \text{ cm}$$

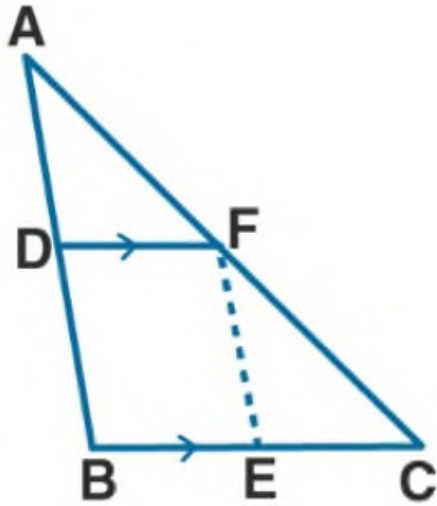
(b) It is given that

D and E are the mid-point of sides AB and AC

$$BC = 5.6 \text{ cm and } \angle B = 72^\circ$$

To find:

- (i) DE
- (ii) $\angle ADE$.



We know that

In $\triangle ABC$

D and E is the mid-point of the sides AB and AC

Using mid-point theorem.

$DE \parallel BC$

$$(i) \quad DE = \frac{1}{2} BC = \frac{1}{2} \times 5.6 = 2.8 \text{ cm}$$

$$(ii) \quad \angle ADE = \angle B \text{ are corresponding angles}$$

It is given that

$$\angle B = 72^\circ \text{ and } BC \parallel DE$$

$$\angle ADE = 72^\circ$$

(c) It is given that

D and E are the midpoints of AB and BC respectively

$DF \parallel BC$ and $AF = 2.6 \text{ cm}$

To find:

- (i) BEF is a parallelogram
- (ii) Calculate the value of AC

Proof:

- (i) In $\triangle ABC$

D is the midpoint of AB and $DF \parallel BC$

F is the midpoint of AC ... (1)

F and E are the midpoints of AC and BC

$EF \parallel AB$... (2)

Here $DF \parallel BC$

$DF \parallel BE$ (3)

Using equation (2)

$EF \parallel AB$

$EF \parallel DB$ (4)

Using equation (3) and (4)

DBEF is a parallelogram

- (ii) F is the midpoint of AC

So we get

$$AC = 2 \times AF = 2 \times 2.6 = 5.2 \text{ cm}$$

2. Prove that the four triangle formed by joining in pairs the mid-point of the sides C of a triangle are congruent to each other.

Solution:

It is given that

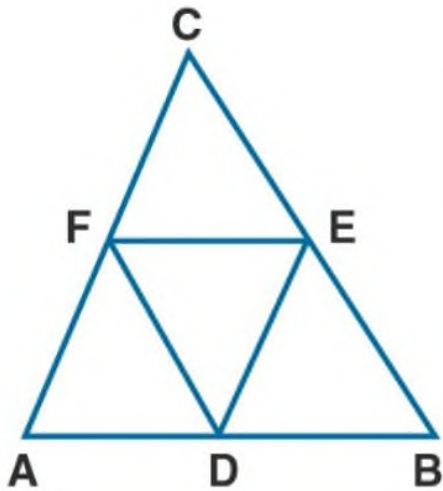
In $\triangle ABC$

D, E and F are the mid-point of AB, BC and CA

Now join DE, EF and FD

To find:

$$\triangle ADF \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF$$



To prove:

In $\triangle ABC$

D and E are the mid-points of AB and BC

$DE \parallel AC$ or FC

Similarly $DF \parallel EC$

DECF is a parallelogram

We know that

Diagonal FE divides the parallelogram DECF in two congruent triangle DEF and CEF

$$\triangle DBE \cong \triangle ECF \quad \dots (1)$$

Here we can prove that

$$\triangle DBE \cong \triangle DEF \quad \dots (2)$$

$$\triangle DEF \cong \triangle ADF \quad \dots (3)$$

Using equation (1), (2) and (3)

$$\triangle ADF \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF$$

3. If D, E and F are mid-point of sides AB, BC and CA respectively of an isosceles triangle ABC, prove that $\triangle DEF$ is also isosceles.

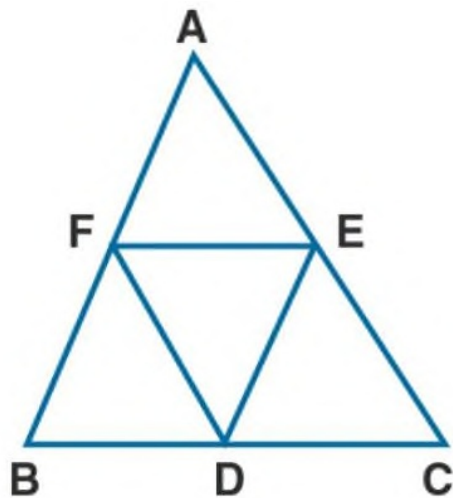
Solution:

It is given that

ABC is an isosceles triangle in which $AB = AC$

D, E and F are the midpoints of the sides BC, CA and AB

Now, D, E and F are joined.



To find:

$\triangle DEF$ is an isosceles triangle

Proof:

D and E are the midpoints of BC and AC.

$$\text{Here } DE \parallel AB \text{ and } DE = \frac{1}{2} AB \quad \dots (1)$$

D and F are the midpoints of BC and AB

Here $DF \parallel AC$ and $DF = \frac{1}{2} AC$... (2)

It is given that

$AB = BC$ and $DE = DF$

Hence, $\triangle DEF$ is an isosceles triangle.

4. The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the midpoint of AD, prove that

(i) $PQ \parallel AB$

(ii) $PO = \frac{1}{2} CD$.

Solution:

It is given that

ABCD is a parallelogram in which diagonals AC and BD intersect each other

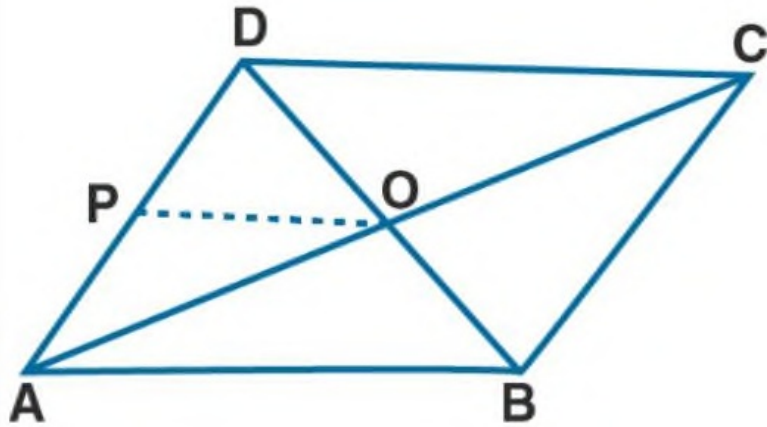
At the point O, P is the midpoint of AD

Join OP

To find:

(i) $PQ \parallel AB$

(ii) $PQ = \frac{1}{2} CD$



Proof:

(i) In parallelogram diagonals bisect each other

$$BO = OD$$

Here O is the midpoint of BD

In $\triangle ABD$

P and O is the midpoint of AD and BD

$$PO \parallel AB \text{ and } PO = \frac{1}{2} AB \quad \dots (1)$$

Hence, it is proved that $PO \parallel AB$.

(ii) ABCD is a parallelogram

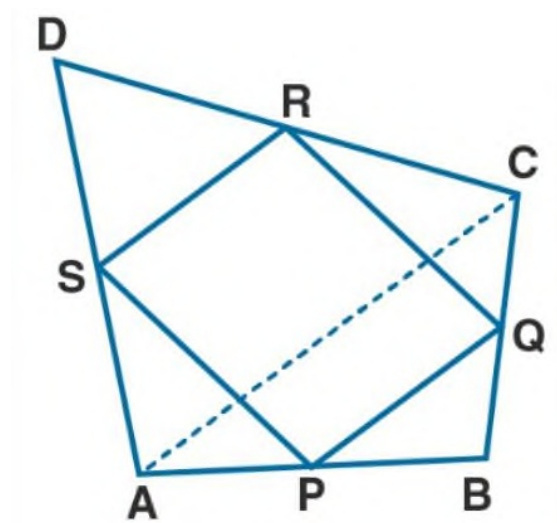
$$AB = CD \quad \dots (2)$$

Using both (1) and (2)

$$PO = \frac{1}{2} CD$$

5. In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-point of AB, BC, CD and DA respectively. AC is its diagonal. Show that

- (i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$
- (ii) $PQ = SR$
- (iii) PQRS is a parallelogram



Solution:

It is given that

In quadrilateral ABCD

P, Q, R and S are the midpoint of sides AB, BC, CD and DA

AC is the diagonal

To find:

- (i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$
- (ii) $PQ = SR$
- (iii) PQRS is a parallelogram

Proof:

- (i) In $\triangle ADC$

S and R are the midpoint of AD and DC

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots (1)$$

Using the mid-point theorem

(ii) In $\triangle ABC$

P and Q are the midpoint of AB and BC

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots (2)$$

Using equation (1) and (2)

$$PQ = SR \text{ and } PQ \parallel SR$$

(iii) $PQ = SR$ and $PQ \parallel SR$

Hence, PQRS is a parallelogram.

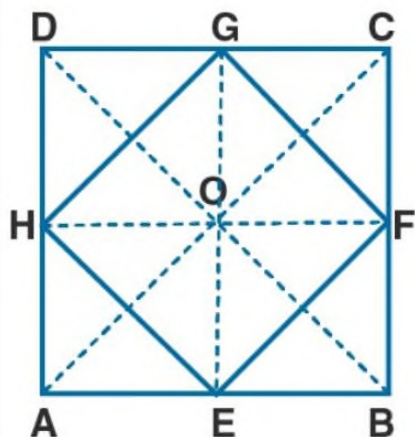
6. Show that the quadrilateral formed by joining the midpoint of the adjacent sides of a square, is also a square.

Solution:

It is given that

A square ABCD in which E, F, G and H are mid-points of AB, BC, CD and DA

Join EF, FG, GH and HE.



To find:

EFGH is a square

Construct AC and BD

Proof:

In $\triangle ACD$

G and H are the midpoint of CD and AC

$$GH \parallel AC \text{ and } GH = \frac{1}{2} AC \quad \dots (1)$$

In $\triangle ABC$, E and F are the midpoint of AB and BC

$$EF \parallel AC \text{ and } EF = \frac{1}{2} AC \quad \dots (2)$$

Using both the equations

$$EF \parallel AC \text{ and } EF = GH = \frac{1}{2} AC \quad \dots (3)$$

In the same way we can prove that

$$EF \parallel GH \text{ and } EH = GF = \frac{1}{2} BD$$

We know that the diagonals of square are equal

$$AC = BD$$

By dividing both sides by 2

$$\frac{1}{2} AC = \frac{1}{2} BD \quad \dots (4)$$

Using equation (3) and (4)

$$EF = GH = EH = GF \quad \dots (5)$$

Therefore, EFGH is a parallelogram

In $\triangle GOH$ and $\triangle GOF$

OH and OF as the diagonals of parallelogram bisect each other

OG = OG is common

Using equation (5)

GF = GF

$\triangle GOH \cong \triangle GOF$ (SSS axiom of congruency)

$\angle GOH = \angle GOF$ (c.p.c.t.)

We know that

$\angle GOH + \angle GOF = 180^\circ$ as it is a linear pair

$\angle GOH + \angle GOH = 180^\circ$

So we get

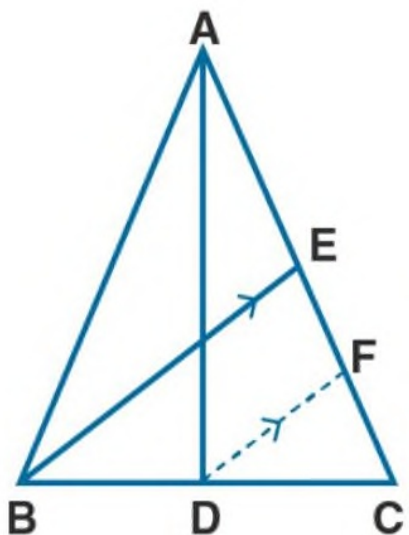
$2\angle GOF = 180^\circ$

$\angle GOF = \frac{180^\circ}{2} = 90^\circ$

So the diagonals of a parallelogram ABCD bisect and perpendicular to each other

Hence, it is proved that EFGH is a square.

7. In the adjoining figure, AD and BE are medians of $\triangle ABC$. If $DF \parallel BE$, Prove that $CF = \frac{1}{2} AC$.



Solution:

It is given that

AD and BE are the medians of $\triangle ABC$

Construct $DF \parallel BE$

To find:

$$CF = \frac{1}{4} AC$$

Proof:

In $\triangle BCE$

D is the midpoint of BC and $DF \parallel BE$

F is the midpoint of EC

$$CF = \frac{1}{2} EC \quad \dots (1)$$

E is the midpoint of AC

$$EC = \frac{1}{2} AC \quad \dots (2)$$

Using both the equations

$$CF = \frac{1}{2} EC = \frac{1}{2} \left(\frac{1}{2} AC \right)$$

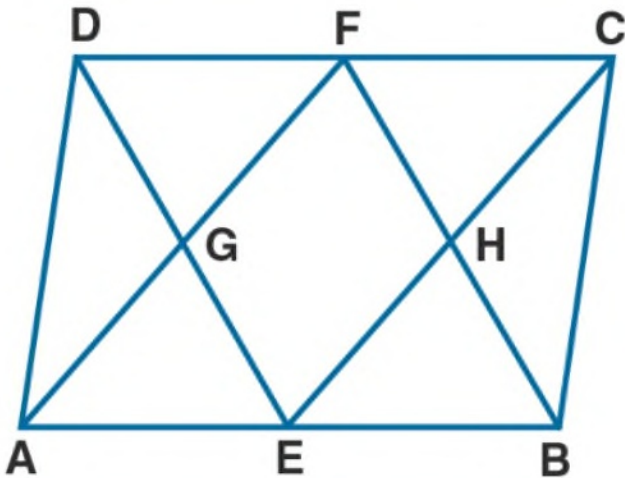
So we get

$$CF = \frac{1}{4} AC$$

Hence, it is proved.

8. In the figure (1) given below, ABCD is a parallelogram. E and F are midpoint of the sides AB and CO respectively. The straight lines ED and EC in points G and H respectively. Prove that

- (i) $\triangle HEB = \triangle HCF$
- (ii) GEHF is a parallelogram.



Solution:

It is given that

ABCD is a parallelogram

E and F are the midpoint of sides AB and CD

To prove:

- (i) $\triangle HEB = \triangle HCF$
- (ii) GEHF is a parallelogram.

Proof:

(i) We know that

ABCD is a parallelogram

$FC \parallel BE$

$\angle CEB = \angle FCE$ are alternate angle

$$\angle HEB = \angle FCH \quad \dots (1)$$

$\angle EBF = \angle CEB$ are alternate angle

$$\angle EBH = \angle CFM \quad \dots (2)$$

Here E and F are midpoint of AB and CD

$$BE = \frac{1}{2} AB \quad \dots (3)$$

$$CF = \frac{1}{2} CD \quad \dots (4)$$

We know that

ABCD is a parallelogram

$$AB = CD$$

Now dividing both sides by $\frac{1}{2}$

$$\frac{1}{2} AB = \frac{1}{2} CD$$

Using equation (3) and (4)

$$BE = CF \quad \dots (5)$$

In $\triangle HEB$ and $\triangle HCF$

$$\angle HEB = \angle FCH \text{ using equation (1)}$$

$$\angle EBH = \angle CFH \text{ using equation (1)}$$

$BE = CF$ using equation (5)

So we get

$\triangle HEB \cong \triangle HCF$ (ASA axiom of congruency)

Hence, it is proved.

(ii) It is given that

E and F are the midpoint of AB and CD

$AB = CD$

So we get

$AE = CF$

Here $AE \parallel CF$

$AE = CF$ and $AE \parallel CF$

So AECF is a parallelogram.

G and H are the midpoint of AF and CE

$GF \parallel EH$ (6)

In the same way we can prove that GFHE is a parallelogram

So G and H are the points on the line DE and BF

$GE \parallel HF$ (7)

Using equation (6) and (7) GEHF is a parallelogram.

Hence, it is proved.

9. ABC is an isosceles triangle $AB = AC$. D, E and F are midpoint of the sides BC, AB and AC respectively. Prove that the line segment AD is perpendicular to EF and is bisected by it.

Solution:

It is given that

ABC is an isosceles triangle with $AB = AC$

D, E and F are mid-point of the sides BC, AB and AC

To find:

AD is perpendicular to EF and is bisected by it.

Proof:

In $\triangle ABD$ and $\triangle ACD$

ABC is an isosceles triangle

$$\angle ABD = \angle ACD$$

Here D is the midpoint of BC

$$BD = CD$$

It is given that $AB = AC$

$\triangle ABD \cong \triangle ACD$ (SAS axiom of congruency)

$\angle ADB + \angle ADC = 180^\circ$ is a linear pair

$$\angle ADB + \angle ADB = 180^\circ$$

By further calculation

$$2\angle ADB = 180^\circ$$

So we get

$$\angle ADB = \frac{180}{2} = 90^\circ$$

So AD is perpendicular to BC (1)

D and E are the midpoint of BC and AB

$DE \parallel AC$ (2)

D and F are the midpoint of BC and AC

$$EF \parallel AD \quad \dots (3)$$

Using equation (2) and (3)

AEDF is a parallelogram

Here the diagonals of a parallelogram bisect each other

AD and EF bisect each other

Using equation (1) and (3)

$$EF \parallel BC$$

So AD is perpendicular to EF

Hence, it is proved.

10.

(a) In the quadrilateral (1) given below, $AB \parallel DC$, E and F are the midpoints of AD and BD respectively. Prove that:

(i) G is midpoint of BC

(ii) $EG = \frac{1}{2}(AB + DC)$

(b) In the quadrilateral (2) given below, $AB \parallel DC \parallel EG$. If E is mid-point of AD prove that:

(i) G is the mid-point of BC

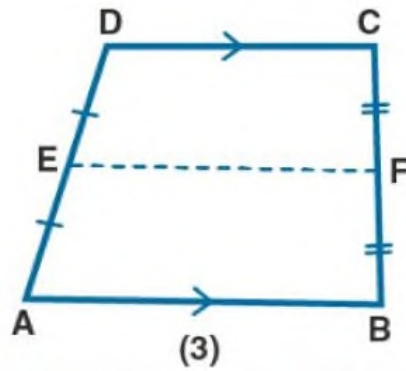
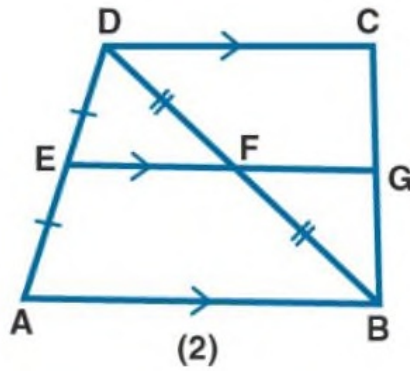
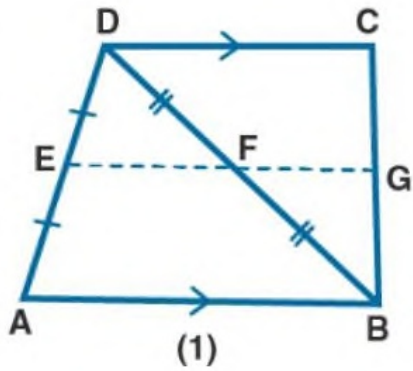
(ii) $2EG = AB + CD$

(c) In the quadrilateral (3) given below, $AB \parallel DC$

E and F are mid-point of non-parallel sides AD and BC respectively. Calculate:

(i) EF if $AB = 6 \text{ cm}$ and $DC = 4 \text{ cm}$.

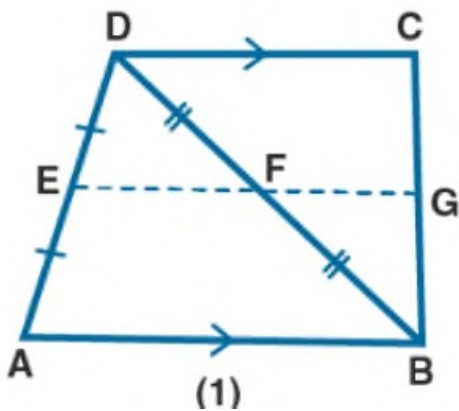
(ii) AB if $DC = 8 \text{ cm}$ and $EF = 9 \text{ cm}$.



Solution:

(a) It is given that

$AB \parallel DC$, E and F are midpoints of AD and BD



To Prove:

(i) G is midpoint of BC

(ii) $EG = \frac{1}{2} (AB + DC)$

Proof:

In $\triangle ABD$

F is the midpoint of BD

$$DF = BF$$

E is the midpoint of AD

$$EF \parallel AB \text{ and } EF = \frac{1}{2} AB \quad \dots (1)$$

It is given that $AB \parallel CD$

$$EG \parallel CD$$

F is the midpoint of BD

$$FG \parallel DC$$

G is the mid-point of BC

$$FG = \frac{1}{2} DC \quad \dots (2)$$

By adding both the equations

$$EF + FG = \frac{1}{2} AB + \frac{1}{2} DC$$

Taking $\frac{1}{2}$ as common

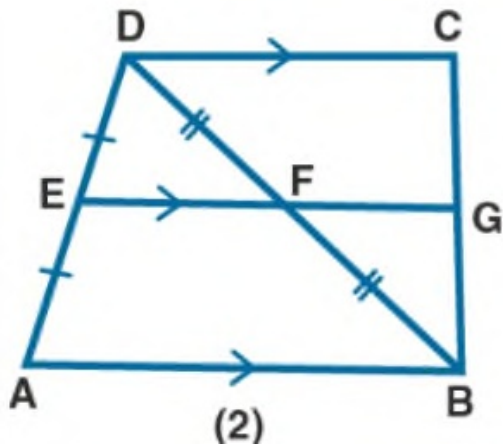
$$EG = \frac{1}{2} (AB + DC)$$

Therefore, it is proved.

(b) It is given that

Quadrilateral ABCD in which $AB \parallel DC \parallel EG$

E is the mid-point of AD



To Prove:

- (i) G is the mid-point of BC
- (ii) $2EG = AB + CD$

Proof:

$AB \parallel DC$

$EG \parallel AB$

So we get

$EG \parallel DC$

In $\triangle DAB$,

E is the midpoint of BD and $EF = \frac{1}{2} AB$ (1)

In $\triangle BCD$,

F is the midpoint of BD and $FG \parallel DC$

$FG = \frac{1}{2} CD$ (2)

By adding both the equations

$$EF + FG = \frac{1}{2} AB + \frac{1}{2} CD$$

Talking out the common terms

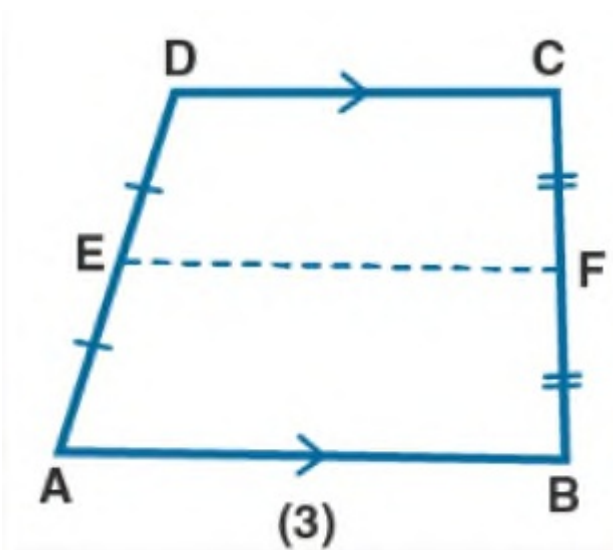
$$EG = \frac{1}{2} (AB + CD)$$

Hence, it is proved.

(c) It is given that

A quadrilateral in which $AB \parallel DC$

E and F are the midpoint of non-parallel sides AD and BC



To Prove

(i) $EF = \frac{1}{2} (AB + DC)$ if $AB = 6$ cm and $DC = 4$ cm.

(ii) $AB = 2EF - DC$ if $DC = 8$ cm and $EF = 9$ cm.

Proof:

We know that

The length of line segment joining the midpoint of two non-parallel sides is half the sum of the lengths of the parallel sides.

E and F are the midpoint of AD and BC

$$EF = \frac{1}{2}(AB + CD) \quad \dots (1)$$

(i) $AB = 6 \text{ cm}$ and $DC = 4 \text{ cm}$

Substituting in equation (1)

$$EF = \frac{1}{2}(6 + 4)$$

By further calculation

$$EF = \frac{1}{2} \times 10 = 5 \text{ cm}$$

(ii) $DC = 8 \text{ cm}$ and $EF = 9 \text{ cm}$

Substituting in equation (1)

$$EF = \frac{1}{2}(AB + DC)$$

By further calculation

$$9 = \frac{1}{2}(AB + 8)$$

$$18 = AB + 8$$

So we get

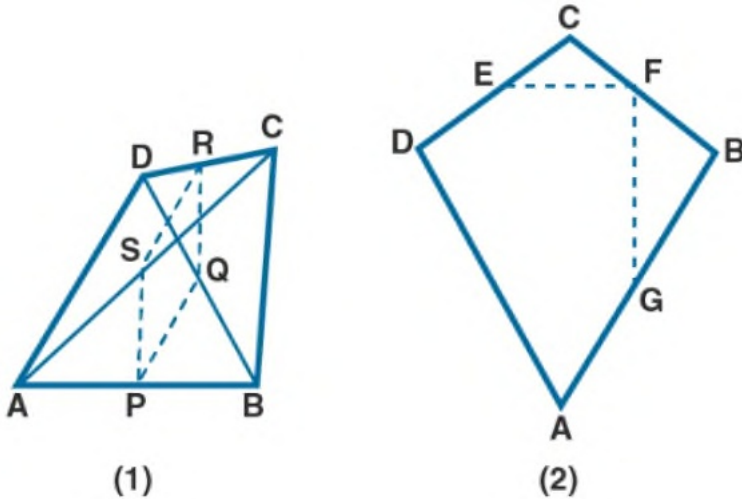
$$18 - 8 = AB$$

$$AB = 10 \text{ cm}$$

11.

- (a) In the quadrilateral (1) given below, $AD = BC$, P, Q, R and S are midpoint of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.
- (b) In the figure (2) given below, ABCD is a kite in which $BC = CD$, $AB = AD$, E, F, G are midpoint of CD, BC and AB respectively. Prove that:

- (i) $\angle EFG = 90^\circ$
(ii) The line drawn through G and parallel to FE bisects DA.



Solution:

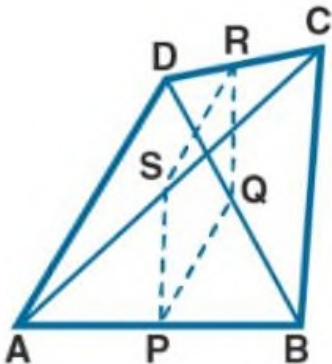
(a) It is given that

A quadrilateral ABCD in which $AD = C$

P, Q, R and S are midpoint of AB, BD, CD and AC

To prove:

PQRS is a rhombus



Proof:

In $\triangle ABD$

P and Q are midpoint of AB and BD

$$PQ \parallel AD \text{ and } PQ = \frac{1}{2} AB \quad \dots (1)$$

In ΔBCD ,

R and Q are midpoint of DC and BD

$$RQ \parallel BC \text{ and } RQ = \frac{1}{2} BC \quad \dots (2)$$

P and S are mid-point of AB and AC

$$PS \parallel BC \text{ and } PS = \frac{1}{2} BC \quad \dots (3)$$

$$AD = BC$$

Using all the equations

$$PS \parallel RQ \text{ and } PQ = PS = RQ$$

$$\text{Here } PS \parallel RQ \text{ and } PS = RQ$$

PQRS is a parallelogram

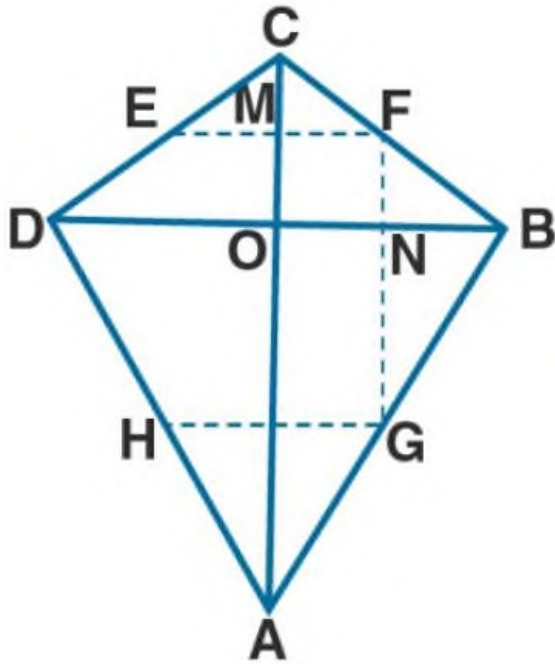
$$PQ = RS = PS = RQ$$

PQRS is a parallelogram

Therefore, it is proved.

(ii) It is given that

ABCD is a kite in which $BC = CD$, $AB = AD$, E, F, G are midpoint of CD, BC and AB



To prove:

- (i) $\angle EFG = 90^\circ$
- (ii) The line drawn through G and parallel to FE bisects DA

Construction:

Join AC and BD

Construct GH through G parallel to FE

Proof:

- (i) We know that

Diagonals of a kite intersect at right angles

$$\angle MON = 90^\circ \quad \dots (1)$$

In $\triangle BCD$,

E and F are midpoint of CD and BC

$$EF \parallel DB \text{ and } EF = \frac{1}{2}DB \quad \dots (2)$$

$EF \parallel DB$

$MF \parallel ON$

Here

$$\angle MON + \angle MFN = 180^\circ$$

$$90^\circ + \angle MFN = 180^\circ$$

By further calculation

$$\angle MFN = 180 - 90 = 90^\circ$$

$$\text{So } \angle EFG = 90^\circ$$

Hence, it is proved.

(ii) In $\triangle ABD$,

G is the midpoint of AB and $HG \parallel DB$

Using equation (2)

$EF \parallel DB$ and $EF \parallel HG$

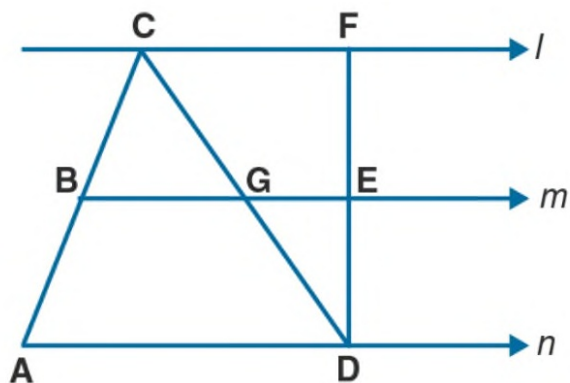
$HG \parallel DB$

Here H is the midpoint of DA

Therefore, the line drawn through G and parallel to FE bisects DA.

12. In the adjoining figure, the lines l, m and n are parallel to each other, and G is midpoint of CD. Calculate:

- (i) BG if AD = 6 cm**
- (ii) CF if GE = 2.3 cm**
- (iii) AB if BC = 2.4 cm**
- (iv) ED if FD = 4.4 cm**



Solution:

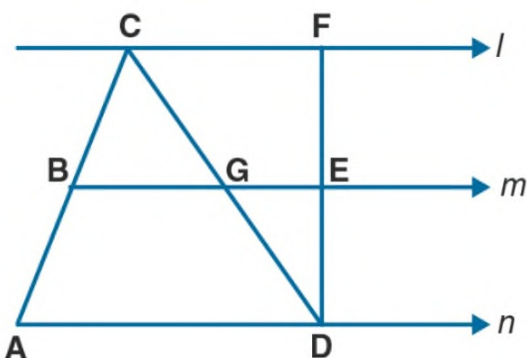
It is given that

The straight line l , m and n are parallel to each other

G is the midpoint of CD

To find:

- (i) BG if $AD = 6$ cm
- (ii) CF if $GE = 2.3$ cm
- (iii) AB if $BC = 2.4$ cm
- (iv) ED if $FD = 4.4$ cm



Proof:

- (i) In $\triangle ACD$,

G is the midpoint of CD

$BG \parallel AD$ as $m \parallel n$

Here B is the midpoint of AC and $BG = \frac{1}{2} AD$

So we get

$$BG = \frac{1}{2} \times 6 = 3 \text{ cm}$$

(ii) In $\triangle CDF$,

G is the midpoint of CD

$GE \parallel CF$ as $m \parallel l$

Here E is the midpoint of DF and $GE = \frac{1}{2} CF$

So we get

$$CF = 2GE$$

$$CF = 2 \times 2.3 = 4.6 \text{ cm}$$

(iii) From (i)

B is the midpoint of AC

$$AB = BC$$

We know that

$$BC = 2.4 \text{ cm}$$

$$\text{So } AB = 2.4 \text{ cm}$$

(iv) From (ii)

E is the mid-point of FD

$$ED = \frac{1}{2} FD$$

We know that

$$FD = 4.4 \text{ cm}$$

$$ED = \frac{1}{2} \times 4.4 = 2.2 \text{ cm.}$$

CHAPTER TEST

1. ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD respectively. Prove that $PQ \perp QR$.

Solution:

It is given that

ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD

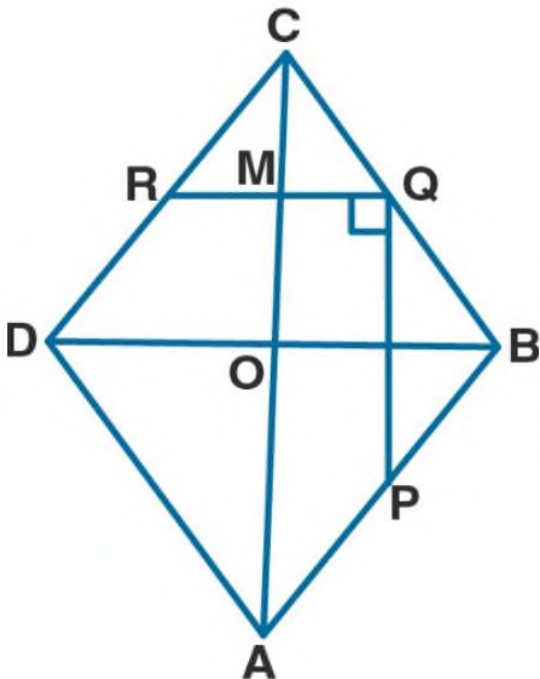
To prove:

$PQ \perp QR$

Construction: Join AC and BD

Proof:

Diagonals of rhombus intersect at right angle



$$\angle MON = 90^\circ \quad \dots (1)$$

In $\triangle BCD$

Q and R are midpoint of BC and CD.

$$RQ \parallel DB \text{ and } RQ = \frac{1}{2} DB \quad \dots (2)$$

Here

$$RQ \parallel DB$$

$$MQ \parallel ON$$

We know that

$$\angle MON + \angle MON = 180^\circ$$

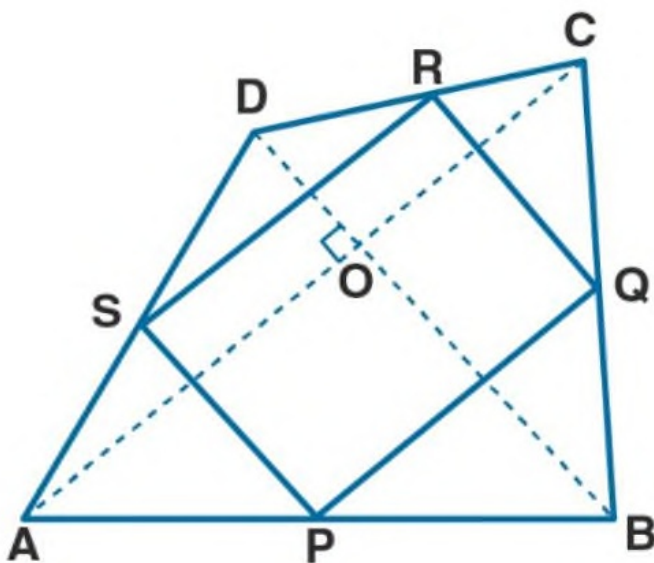
$$\angle MON = 180 - 90 = 90^\circ$$

So $NQ \perp MQ$ or $PQ \perp QR$

Hence, it is proved.

2. The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the midpoint of its adjacent sides is a rectangle.

Solution:



It is given that

ABCD is a quadrilateral in which diagonals AC and BD are perpendicular to each other

P, Q, R and S are mid-point of AB, BC, CD and DA

To prove:

PQRS is a rectangle

Proof:

We know that

P and Q are the midpoint of AB and BC

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad \dots (1)$$

S and R are midpoint of AD and DC

$$SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad \dots (2)$$

Using both the equations

$$PQ \parallel SR \text{ and } PQ = SR$$

So PQRS is a parallelogram

AC and BD intersect at right angles

$$SP \parallel BD \text{ and } BD \perp AC$$

So $SP \perp AC$ i.e. $SP \perp SR$

$$\angle RSP = 90^\circ$$

$$\angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^\circ$$

Hence, PQRS is a rectangle.

3. If D, E, F are midpoint of the sides BC, CA and AB respectively of a $\triangle ABC$, prove that AD and FE bisect each other.

Solution:

It is given that

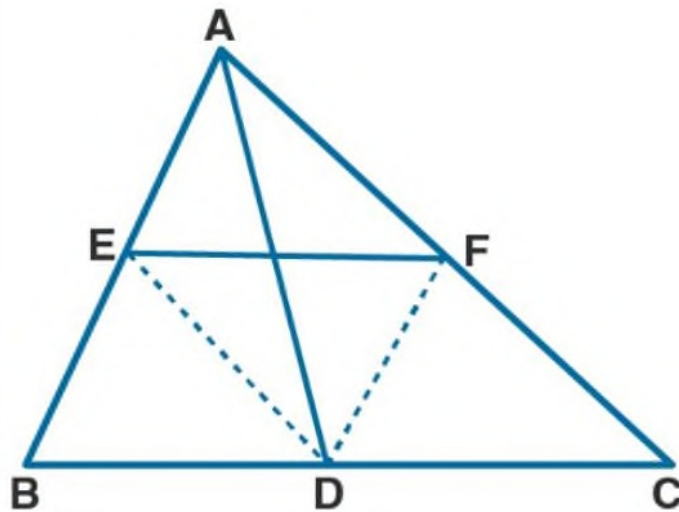
D, E, F are midpoint of sides BC, CA and AB of a $\triangle ABC$

To prove:

AD and FE bisect each other

Construction:

Join ED and FD



Proof:

We know that

D and E are the midpoint of BC and AB

$DE \parallel AC$ and $DE \parallel AF$... (1)

D and F are the midpoint of BC and AC

$DF \parallel AB$ and $DF \parallel AE$... (2)

Using both the equations

ADEF is a parallelogram

Here the diagonals of a parallelogram bisect each other

AD and EF bisect each other.

Therefore, it is proved.

4. In $\triangle ABC$, D and E are midpoint of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If $AB = 8$ cm and $BC = 9$ cm, find the perimeter of the parallelogram BDEF.

Solution:

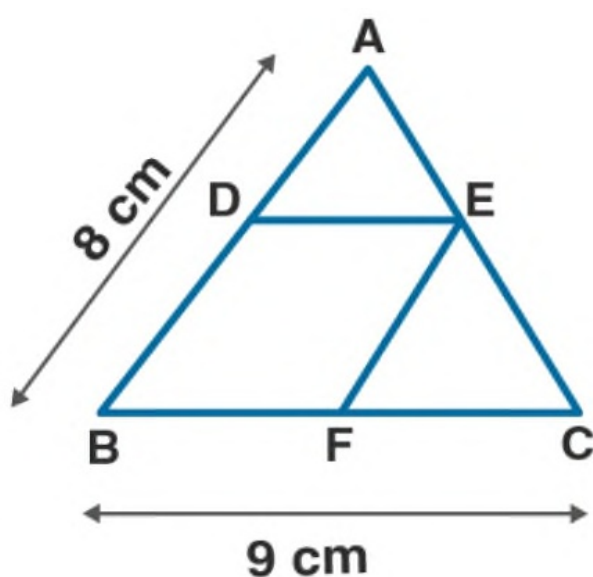
It is given that

In $\triangle ABC$

D and E are the midpoint of sides AB and AC

DE is joined from E

$EF \parallel AB$ is drawn $AB = 8$ cm and $BC = 9$ cm



To Prove:

- (i) BDEF is parallelogram
- (ii) Find the perimeter of BDEF

Proof:

In $\triangle ABC$

B and E are the mid-point of AB and AC

Here $DE \parallel BC$ and $DE = \frac{1}{2} BC$

So $EF \parallel AB$

DEFB is a parallelogram

$DE = BF$

So we get

$$DE = \frac{1}{2} BD = \frac{1}{2} \times 9 = 4.5 \text{ cm}$$

$$EF = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4 \text{ cm}$$

We know that

$$\text{Perimeter of BDEF} = 2 (DE + EF)$$

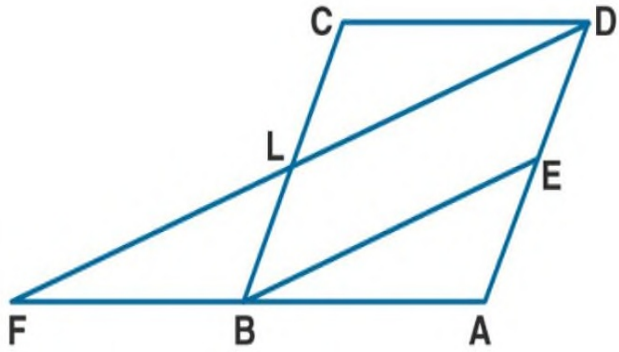
Substituting the value

$$= 2(4.5 + 4)$$

$$= 2 \times 8.5$$

$$= 17 \text{ cm}$$

5. In the given figure, ABCD is a parallelogram and E is mid-point of AD, DL $\parallel$ EB meets AB produced at F. Prove that B is midpoint of AF and EB = LF.



Solution:

It is given that

ABCD is a parallelogram

E is the midpoint of AD

$DL \parallel EB$ meets AB produced at F

To prove:

$$EB = LF$$

B is the midpoint of AF

Proof:

We know that

$BC \parallel AD$ and $BE \parallel LD$

BEDL is a parallelogram

$$BE = LD \text{ and } BL = AE$$

Here E is the midpoint of AD

L is the midpoint of BC

In $\triangle FAD$

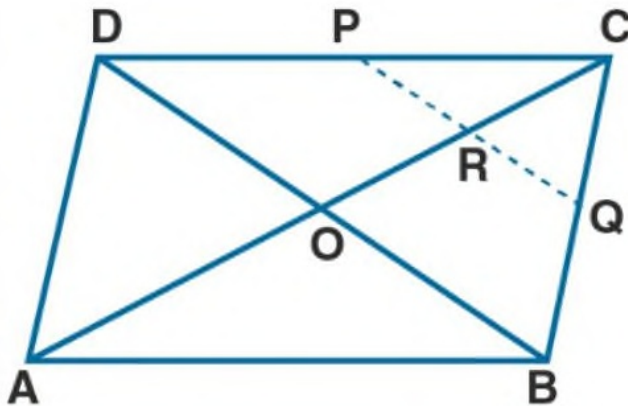
E is the mid-point of AD and $BE \parallel LD$ at FLD

So B is the midpoint of AF

Here

$$EB = \frac{1}{2} FD = LF.$$

6. In the given figure, ABCD is a parallelogram. If P and Q are mid-point of sides CD and BC respectively, show that $CR = \frac{1}{2} AC$.



Solution:

It is given that

ABCD is a parallelogram

P and Q are midpoint of CD and BC

To prove: $CR = \frac{1}{4} AC$

Construction: Join AC and BD

Proof:

In parallelogram ABCD

Diagonals AC and BD bisect each other at O

$$AO = OC \text{ or } OC = \frac{1}{2} AC \quad \dots (1)$$

In $\triangle BCD$

P and Q are midpoint of CD and BC

To prove: $CR = \frac{1}{4} AC$

Construction: Join AC and BD

Proof:

In parallelogram ABCD

Diagonals AC and BD bisect each other at O

$$AO = OC \text{ or } OC = \frac{1}{2} AC \quad \dots (1)$$

In ΔBCO

Q is the midpoint of BC and $PQ \parallel OB$

Here is the midpoint of CO

So we get

$$CR = \frac{1}{2} OC = \frac{1}{2} \left(\frac{1}{2} AC \right)$$

$$CR = \frac{1}{4} AC$$

Hence, it is proved.